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Ancient Greek Sundials and the Theory of Conic Sections Reconsidered

Summary

In this article, a new aspect of a possible connection between the development of ancient Greek and Roman sundials and the history of conic sections is analyzed: On conical sundials, a conic section occurs at the edge between the plane top surface and a conical surface which is most commonly used in sundials as the shape of the dial face. Based on an analysis of 3D models of conical sundials, this paper argues that the curve is not the result of a method of shaping the conical surface, but rather the basis to do so. A method is given by which the curve can be drawn approximately by connecting points. The latter can be found using a geometrical construction. This procedure suggests that craftsmen who built sundials had basic knowledge of the geometry of cones and conic sections.

Keywords: Ancient sundial; conic section; theory of conic sections; manufacturing; history of mathematics; history of technology; 3D model.

Im Fokus steht ein neuer Aspekt der möglichen Verbindung zwischen der Entwicklung antiker griechischer und römischer Sonnenuhren und der Geschichte der Kegelschnitte: Bei konischen Sonnenuhren tritt an der Kante zwischen der ebenen Oberseite und einer kegelförmigen Fläche, der häufigsten Variante der Schattenfläche, ein Kegelschnitt auf. Basierend auf einer Analyse von 3D-Modellen konischer Sonnenuhren wird dafür argumentiert, dass die Kurve nicht das Resultat einer Methode der Ausführung der Fläche ist, sondern ihre Grundlage. Eine Methode zum approximativen Zeichnen der Kurve wird angegeben, bei der geometrisch konstruierte Punkte verbunden werden. Die Prozedur legt nahe, dass Handwerker der Sonnenuhrenherstellung Grundkenntnisse der Geometrie des Kegels und der Kegelschnitte besaßen.

Keywords: Antike Sonnenuhr; Kegelschnitt; Theorie der Kegelschnitte; Fertigungsprozess; Mathematikgeschichte; Technikgeschichte; 3D-Modell.

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1 Introduction

It is neither new nor ill-founded to think about the connection between the origin of ancient Greek sundials and the history of the theory of conic sections.¹ Besides the quite obvious presence of conic sections as the lines of the daily motion of the shadow of the tip of a gnomon on a plane surface that are usually represented on planar sundials for some special days in the year, such lines appear in a completely different context in one other type of ancient Greek sundials: For the largest group of sundials, the shadow receiving surface is part of the surface of a right cone whose axis directs towards the poles. Since all those sundials have a plane top surface that is parallel to the horizon, the intersection of those two surfaces is a non-circular conic section. But has this curve been recognized as such? This analysis aims to reconstruct the role of this conic section in the geometry of conic sundials and its relation to Greek theories of conic sections. Contrary to earlier approaches, this analysis is based on an evaluation of the material evidence.

2 The geometry of conical sundials

With the exception of a small group from the islands of Rhodes and Kos, and a handful of other objects, almost all sundials with conical shadow receiving surface share the same design. They consist of a stone block whose south facing front face is divided into two parts. Whereas the upper part is always a plane surface that is inclined relative to the plane top side, the lower part can have different layouts. In many cases there is another inclined plane surface which intersects the upper plane in a straight line parallel to the other intersections of the top, back, and bottom planes. Often, the planar part of the lower section of the front face is decorated with two lion paws on its left and right sides or stylized feet-like elements on the same position as in a sundial from Delos (Fig. 1).

¹ For example, in Neugebauer 1948 and Neugebauer 1975, 857, Otto Neugebauer suggests that the early

theory of conic section originated from the theory of sundials.



Fig. 1 View on the south and west (left), and on the top side (right) of a conical sundial from Delos. Archaeological Museum Delos, Inv. B3652 (11023).

The stone block is intersected by a right cone that stands orthogonal on the upper part of the front face. On its surface, eleven hour lines – and usually three lines that show the daily motion of the sun for some days of the year – are marked. This grid of lines makes it possible to determine the time that is indicated by the shadow of the tip of the gnomon that is situated in the top surface. In order to show the right time, the upper part of the front face and the planes of the day curves have to be parallel to the equatorial plane.

Conical sundials make up the largest group (about 35%) among the preserved sundials of the Greek type.² The oldest sundials with a design as described above date to the beginning of the 2nd century BCE. Some conical sundials with different corpus forms are even slightly older.³ Conical sundials therefore belong to the earliest sundials that have come down to us.

An evaluation of 3D models of sundials shows that the conical surfaces deviate only little from right cones and are indeed orthogonal to the inclined upper part of the front face.⁴ Due to the relative positions of cone and block, the edge between the conical surface and the planar top surface is a conic section. On the preserved objects we can observe three different types of curves: ellipses, parabolas, and hyperbolas (Fig. 2).

An analysis of a group of conical sundials from the island of Delos has shown that there exists a single principle that can explain some key dimensions of those objects.

2 This value is based on information on the preserved objects given in reports on sundials and catalogs such as Gibbs 1976 and Schaldach 2006.

3 For example, a sundial found at Herakleia with two conical dialfaces at the south and the north facing sides dates to the 3rd century BCE (see Berlin Sun-

dial Collaboration 2014a and Berlin Sundial Collaboration 2014b).

4 These properties of the conical sundial type have been suggested by earlier scholars such as Sharon L. Gibbs (Gibbs 1976) but have never been shown before. In many cases the analysis of the geometry of sundials is based on the assumption of these properties.

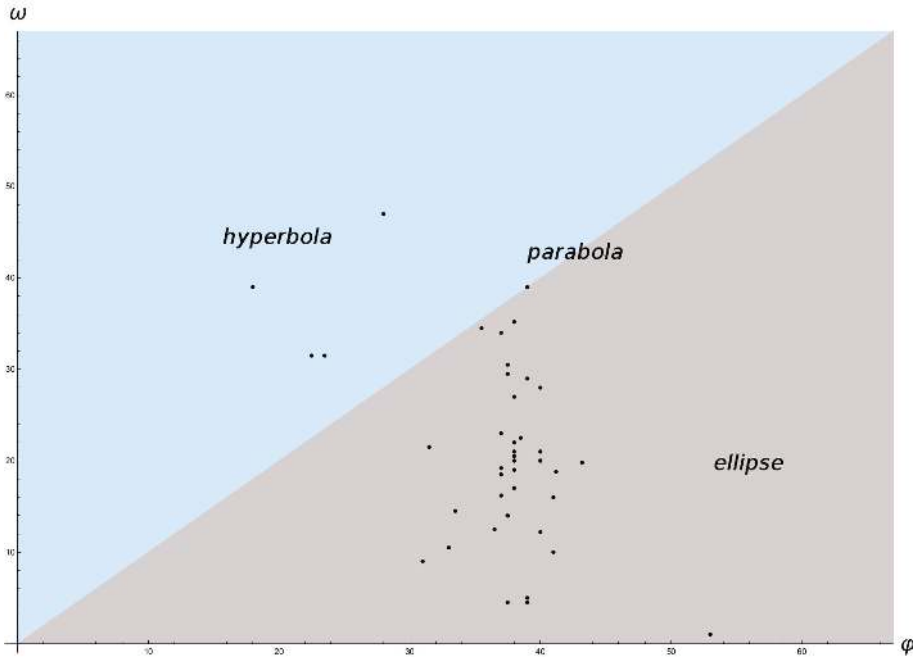


Fig. 2 Type of conic sections at the intersection of the cone with the planar top surface. Depending on the inclination of the front face towards the vertical direction (φ) and the angle at the top opening (ω), the conic section is an ellipse, parabola or a hyperbola. The information used for the diagram is the result of a survey of literature on ancient sundials such as Gibbs 1976 and Schaldach 2006 as well as reports on ancient sundials.

Starting with the geographical latitude of the place given by the ratio of the length of a gnomon g_0 to the length s_0 of its equinoctial shadow the lengths of a set of edges can be derived by easy calculations (Fig. 3):

		number of modular units
r	radius	s_0
g		g_0
s		s_0
w	width	$2 \cdot s_0 + 1$
h	height	$2 \cdot s_0 + 1$
d	depth	$s_0 + 3$
l	depth of cone opening	$s_0 + 1$

Besides one missing parameter and a decision about the design of the base part of the sundial, the shape of the sundial including its cone is determined uniquely by this principle.

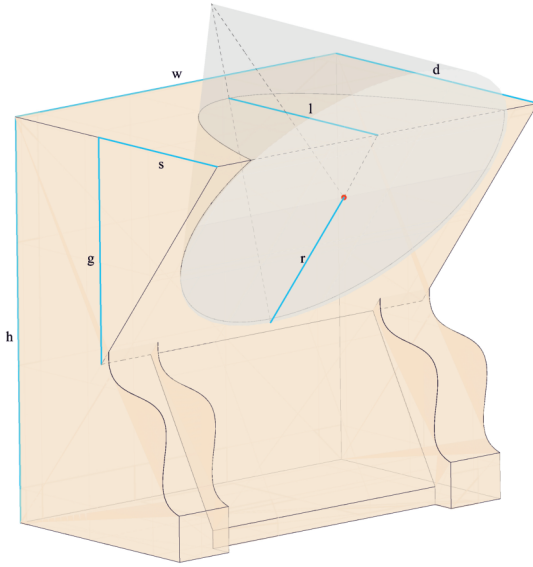


Fig. 3 Dimensions of conical sundials that are determined by a constructional principle found for a group of sundials from Delos. w : width; h : height; d : depth; r : radius of the opening circle; g and s : sides of the orthogonal triangle given by the inclined front side; l : distance of the point of the conic section that is most distant to the front edge to this edge.

The principle leads – at least for the shaping of the corpus – to an easy to follow procedure that enables the stonecutter to build a sundial for a given geographical latitude without knowledge of astronomy and geography, or a deep understanding of their geometry. Traces of the use of this procedure to determine the position of the upper inclined front side can be identified on several preserved sundials from Delos and elsewhere.

3 Shaping the conical surface

By the principle derived from the Delian sundials, the shape of the cone is almost⁵ given. We still have to identify the craftsman's method for creating the conical surface on the object. Since a large part of the circular base of the cone is given, in a first attempt one might consider a method based on one possible definition of cones: The surface of a cone is generated by the rotation of a straight line fixed at a point (the vertex of the cone) around the circle at its base. Since the cone should be a right cone, the vertex lies

5 The missing parameter determines the position of the center of the opening circle of the sundial. Due to the geometrical properties and the condition to

meet the restrictions coming from the obliquity of the ecliptic, the possible variation in its position is very small.



Fig. 4 Deviations of the cone axes towards the pole direction for well preserved conical sundials, based on the existing 3D models. The distances to the center indicates the deviation in the inclination, the directions of the arrows show their directions relative to the horizon and meridian line.

on the line orthogonal to its base circle standing on its middle point. The shaping of the conical shadow receiving surface then can be controlled by the use of a ruler that is fixed on the vertex of the cone. The position of the vertex can be constructed on the basis of the dimensions given by the construction principle.

As a result of this method one would expect deviations in the fixed point of the ruler from the vertex in any direction. This would lead to deviations of the angles of the cone's axes to the upper part of the corresponding front planes from a right angle in any direction. But this is not in accordance with the material evidence: In most cases, the direction of the cone axis lies within the meridian plane (Fig. 4). So what we seek is a method that can explain this very specific error.

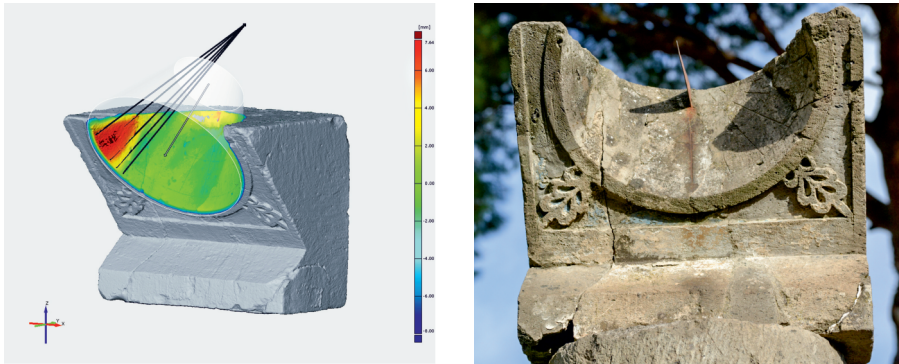


Fig. 5 Left: deviation of the shadow receiving surface towards an exact right cone in a conical sundial from the Villa San Marco, Castellammare di Stabia, Italy, based on its 3D model (Berlin Sundial Collaboration 2015a). Right: photo of the conical sundials from Villa San Marco, Castellammare di Stabia, Italy.

A different approach is to use both the circular line on the upper part of the front side and the line of the conic section that results from the intersection of the cone with the top plane. Again, the removal of the material can be controlled by a ruler. The surface is reached, if all material which lies between any two points of the two given lines is removed. Due to the convexity of cones no part of those lines falls outside the cone. Since the convex hull of the two curves contains the section of the cone's surface we know that the method leads to a removal of all redundant material. This method is more robust regarding deviations of the cone's axis to the west and east than to directions lying in the meridian plane. By this, we can explain the characteristics of the errors as shown in Fig. 4.

Deviations of the resulting cone then can be caused by drawing incorrect conic sections on the top surface, errors in the usage of the dimension of the distance of the deepest point of the cone to the front edge l , erroneous inclinations of the upper part of the front side towards the top surface, errors in the circle on the front side, or by stopping the process before the conic surface is reached.

The effect of those errors can be observed in the preserved sundials. For example, on a sundial from the Villa San Marco at Castellammare di Stabia, the western part of the conical surface does not coincide with the circular line that is engraved on the front side (Fig. 5 right). In this area, the surface of the object deviates from the exact conic surface (Fig. 5 left). Going to the top surface, this deviation becomes smaller.

This situation can be explained by the use of the method based on the conic section on the top surface, if the process has been stopped before the conic surface has been reached. Of course it could be generated by an erroneous usage of the first method. But in this case the vertex had to be moving, the circle on the front side has not been met,

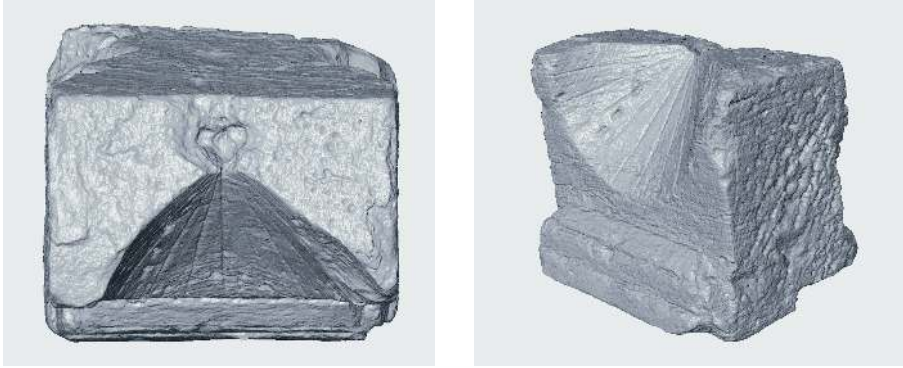


Fig. 6 View on the top plane (left) and the south eastern edge (right) of the sundial from Naga, based on its 3D model (TrigonArt, Bauer Praus GbR 2015 [2013]).

and the procedure stopped when – by chance – the correct conic section at the top plane has been reached. This is not plausible.

In the case of the Naga sundial⁶ the curve at the top plane deviates from a conic section (Fig. 6 left). Nevertheless the surface is generated by an (erroneous) sequence of straight connections of points lying on both defining curves. In the relevant part of the surface the direction of these lines is the same as in the markings that come from the carving of the surface and are preserved at the western part of the surface (Fig. 6 right). Again, we see the result of the second method, based on erroneous starting conditions.

Since the method based on the conic section can explain the very specific errors observed on the material evidence, it has to be considered as the one that has been used to build those sundials.

4 Greek mathematics and the method of shaping the cone

The first Greek mathematical texts dealing with the geometry of cones – especially right cones – significantly predate the first preserved sundials. About the time of the earliest preserved conical sundials Apollonius of Perga states two propositions at the very beginning of his *Conics*:

Prop. 1: The straight lines drawn from the vertex of the conic surface to points on the surface are on that surface.

⁶ Kroeper and Krzyzaniak 1998.

Prop. 2: If on either one of the two vertically opposite surfaces two points are taken, and the straight line joining the points, when produced, does not pass through the vertex, then it will fall within the surface, and produced it will fall outside.⁷

As a consequence, all straight lines connecting the circle and the conic section as in the situation of the second method will fall inside or on the conical surface. So, using two very elementary propositions it can be proved that the method meets the demand.⁸ Moreover, this illustrates that at least within the context of Greek geometry of the time people were aware of this central aspect of the reconstructed method. This shows that this part of the method is historically adequate.

In the light of the theory of conic sections as presented by Apollonius, the curve at the top plane of a conical sundial is a conic section. Depending on its properties one would have to call it an ellipse, parabola or hyperbola.

But since the top plane is not orthogonal to any of the straight lines of the conical surface, according to the reconstruction of the older theory of conic sections the curve is not considered as a representative of one of the three types of conic sections – section of an acute-angled cone, section of a right-angled cone, and section of an obtuse-angled cone – that are analyzed and used in the geometry based on this theory. Nevertheless, some statements suggest a broader use of those terms for all curves that can be generated by an orthogonal intersection of a plane with a cone,⁹ but the terms themselves are still used for example by Archimedes in his *On Conoids and Spheroids* shortly before the time of Apollonius' *Conics*.

Both the first conical sundials and the change in the concept fall into the same time. So what we know is that some people were aware of those curves and their properties, but the lines might not have been named as conic sections. Again, this shows that the usage of those sections of planes and cones is in accordance with what we know about the history of mathematics.

⁷ Translation from Taliaferro and Fried 2013.

⁸ Since a large section of the base circle and its intersection with the part of a conic section at the endpoints of those lines is given, the generation of the surface follows.

⁹ This interpretation has been suggested by Heath 1921, 439. One example is found in the introduction of Euclid's *phaenomena* (Berggren and Thomas 1996).

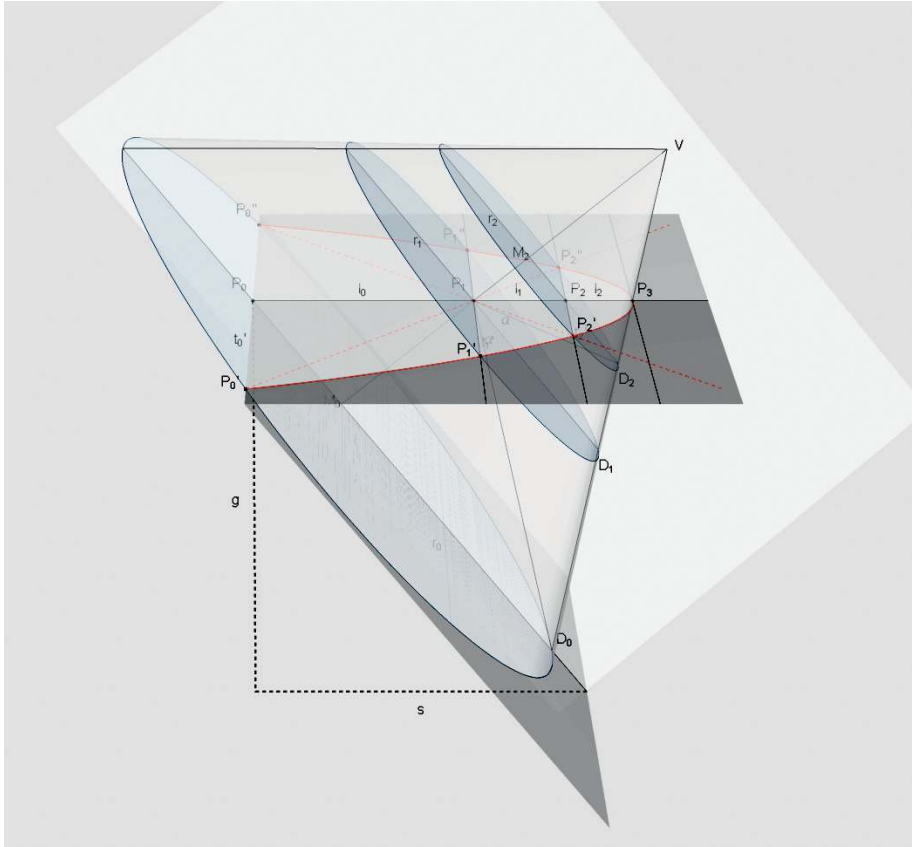


Fig. 7 Geometry of a situation in a conical sundial that can be used as a basis for the construction of 7 supporting points.

5 Methods of drawing the conic section on the top surface

Regardless, whether one calls the edge of the conical shadow receiving surface with the top plane of the sundial a conic sections or not, in order to shape the surface in the identified way one needs a method to find that curve.

Since in most but not all cases the conic section is an ellipse, the method has either to be case sensitive on the type of conic section, or it has to provide the result independent of the type. Due to the spatial limitations on the top surface of the sundial there might be some additional restrictions to the method – unless the conic section is transferred to the object from a separate diagram.

One possibility to do so is based on the following properties of the geometry of a conical sundial (Fig. 7). In a sundial with a right cone that is orthogonal to the upper

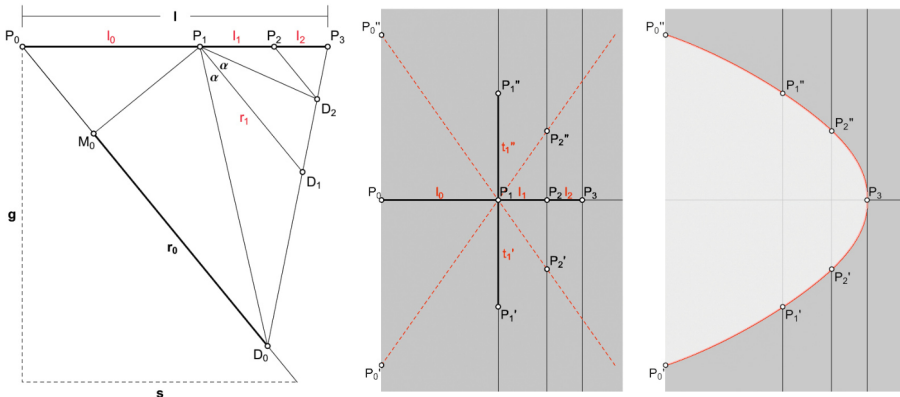


Fig. 8 Analemma-like diagram (left), construction on the top plane (middle), and connection of the supporting points (right) of a method for drawing an arbitrary conic section on a conical sundial.

part of the front face, each plane parallel to the upper part of the front face intersects the cone in a circle. Let two such circles be given. The first should contain the intersection point P_1 of the cone's axis with the top plane,¹⁰ the second circle should be taken such that the angles $D_0P_1D_1$ and $D_1P_1D_2$ are equal. Since in the first plane the middle point P_1 of the intersection circle lies on the top plane, the distances P_1P_1' , P_1P_1'' , and P_1D_1 are all equal to the radius r_1 of this circle. In the given situation, the points P_0' , P_1 , and P_2'' , as well as P_0'' , P_1 , and P_2' , lie on a straight line. This is similar to the situation of the rising and setting of the sun on the solstices and the equinoxes that is considered in the analemma diagram. All geometrical properties can be shown by the use of very basic knowledge of the geometry of right triangles and proportions.

For a sundial for which the inclination of the upper part of the front face, the position and radius of the opening circle, and the inclination of the meridian line on the conical surface are given, one can construct a diagram similar to the analemma diagram (Fig. 8 left). In this situation, the positions of P_1 and D_1 can be derived from the given properties. D_2 and P_2 then can be constructed such that the angles $D_0P_1D_1$ and $D_1P_1D_2$ are equal. From this diagram one can measure the distances l_0 , l_1 , l_2 , and the radius r_1 .¹¹

10 This point is usually suggested as the supposed position of the tip of the gnomon. Since in many objects significant deviations between this point and the plane of the equinoctial day curve can be observed, there might be another type with deviating gnomon point positions.

11 'Meridian line' relates to both the intersection of the meridian plane with the conical surface (as in this context) and the intersection of this plane with the planar top surface.

Using this information together with the geometrical properties stated above gives the possibility to construct seven points of the conic section (Fig. 8 middle):

- Points P'_0 and P''_0 are given by the intersection of the opening circle of the cone on the upper part of the front face and the edge between the upper front and top plane
- Points P'_1 and P''_1 lie on a line which is parallel to the front edge with the distance l_0 as constructed in the diagram in Fig. 8 left. Since as in the 3D diagram both points and point D_1 lie on the same circle with middle point P_1 the distances of points P'_1 and P''_1 to the meridian line on the top plane are equal to the radius r_1 as constructed in diagram in Fig. 8 left.
- Points P'_2 and P''_2 lie on straight lines through point P''_0 (P'_0) and point P_1 . Their distance to the front edge is given in the left diagram (distance $l_0 + l_1$).
- Point P_3 lies on the meridian line on the top plane with distance l as in the left diagram.

This construction works independently of the type of conic section of the curve. By this, one does not need to know the type of the curve for finding positions of some of its points or even have a concept of conic sections.

In a last step, the intersection line of cone and plane can be found – at least in a close approximation – by drawing a smooth curve connecting those points (Fig. 8 right).

A very late witness for drawing a conic section by connecting a number of given points is C. Ptolemy in his *Geography*. At one point,¹² he reminds the reader to care about the correct shape of the ellipses in the depiction of the globe within a ringed sphere/astrolabe.

In prop. 25 of the IVth book of his *Conics*, Apollonius shows that

A section of a cone does not cut a section of a cone or circumference of a circle at more than four points.¹³

According to this, 5 given points suffice to determine the conic section uniquely – as long as the interpolated curve is a conic section. So, the construction provides two additional points. This makes it easier to find the right location of the curve.

The distribution of the given points is a source of errors in the shape of the curve. Since there are only few points in the middle, there is not much guidance in this part. This lack of guidance cannot prevent errors as for example in the Naga sundial: the shape of the conic section is too cuspid in its middle (Fig. 6 left).

12 C. Ptolemy, *Geography*, Book 7, ch. 6.

13 Translation from Taliaferro and Fried 2013.

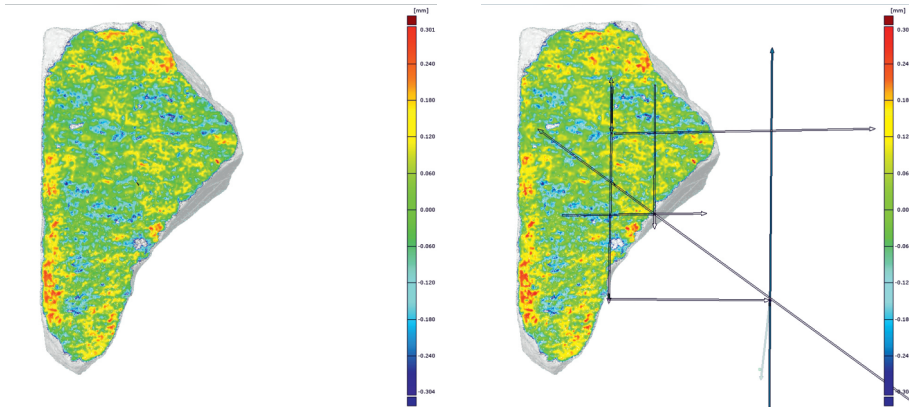


Fig. 9 Constructional lines (right) and their positions (left) on a sundial from Delos (Archaeological Museum, Delos, Inv. B4669) in the 3D model of the object (Berlin Sundial Collaboration 2015b).

Again, for this method we have a similar situation as for the shaping of the conical surface: on one hand, there is a procedure that uses easy geometric constructions to provide everything that is needed to find the correct conic section. On the other hand, there is mathematical knowledge that can prove that the procedure leads to a good result – and that this was known.

There is at least one ancient sundial that shows a very specific set of constructional lines on the top plane that can be part of the construction of those points. Unfortunately, only a fragment of the western part of the upper half of the sundial is preserved. The dimensions of this sundial can be reconstructed according to the principle of the Delos sundials.

All constructional lines are parallel to the intersection of the top and front planes (Fig. 9). One goes through the point most distant to the front edge. The two others are close to the markings of the equinoctial and winter solstitial plane (Fig. 10). The lines are intersected by two other lines that are both orthogonal to the former lines. Two of the intersection points lie very exactly on the now damaged conic section. The line through the intersection point next to the winter solstitial plane and the intersection of the first line with the meridian line on the top plane meets the eastern intersection point of the cone with the front edge and can be seen as constructional line in the back part of the top plane (Fig. 9 left).

A similar situation can be observed on other sundials. What is special in this case is that the foremost constructional line does not coincide exactly with the intersection line of the equatorial plane with the top plane. Otherwise, the lines could also be used to find the location of those planes.

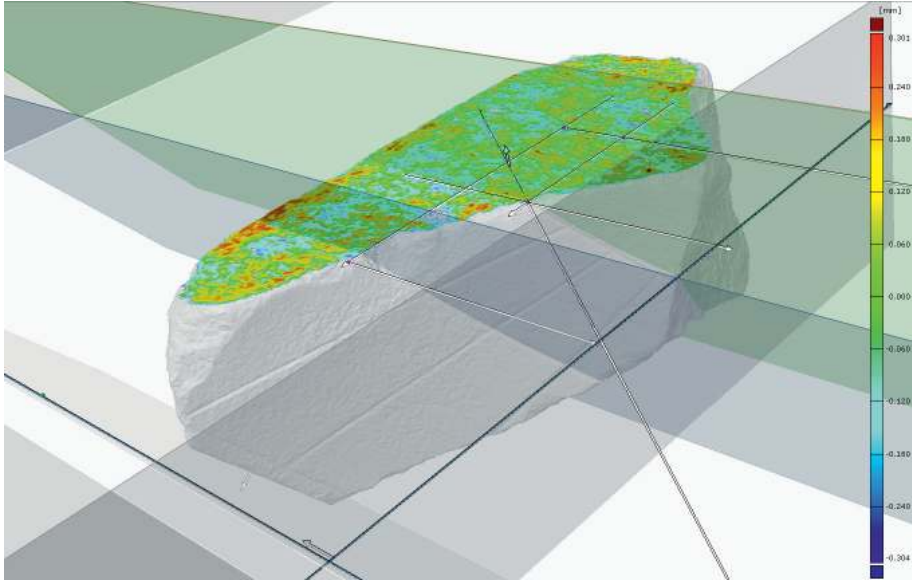


Fig. 10 Constructional lines and their positions relative to the equinoctial and solstitial plane on a sundial from Delos (Archaeological Museum, Delos, Inv. B4669) in the 3D model of the object (Berlin Sundial Collaboration 2015b).

6 Conical sundials and the theory of conic sections

All in all, the conic section that occurs at the intersection of the planar top surface and the conical shadow receiving surface of a conical sundial is not only the result of the geometric configuration of conical sundials. It is crucial in the process of their making. By this, in addition to their contribution to the functionality in other types of sundials, conic sections are of great importance for the design.

The usage of conical shadow receiving surfaces can be traced back into the time of Apollonius of Perga and to a change in the theoretical concepts on conic sections. Whereas the curves that occur in the conical sundials tend to belong to the Apollonian ‘universum’ of conic sections, the mathematics used in the method for shaping the cone are not specific for this author. The generation of both the conic section and the conic surface can be justified with geometrical properties of cones. Even the number of points that are needed to determine the conic section uniquely or at least this question – according to a common interpretation of Apollonius’ own words¹⁴ – goes back to Conon.

14 See also Fried’s introduction to Apollonius, *Conics*, Book IV, in: Taliaferro and Fried 2013, 269.

An influence of the methods of constructing conical sundials to the development of the theory of conic sections has not been found.

Mathematics are not only the means by which the correctness of the outcome of a method for building a sundial can be justified. Advanced geometry is also part of the procedure to cut the stone:

construction of the supporting points of the conic section: needs a geometrical construction similar to the analemma diagram that has to be transferred onto the stone;

drawing of the conic section: requires knowledge of the shapes of conic sections;

shaping of the conical surface: requires knowledge of the shapes of cones.

This suggests that elementary knowledge on cones and conic sections was part of the background of craftsmen who built sundials in ancient Greece.

Bibliography

Berggren and Jones 2000

John Lennart Berggren and Alexander Jones. *Ptolemy's Geography: An Annotated Translation of the Theoretical Chapters*. Princeton and Oxford: Princeton University Press, 2000.

Berggren and Thomas 1996

John Lennart Berggren and Robert S. D. Thomas. *Euclid's Phaenomena: A Translation and Study of a Hellenistic Work in Spherical Astronomy*. Sources and Studies in the History and Philosophy of Classical Sciences 4. New York: Garland Pub., 1996.

Berlin Sundial Collaboration 2014a

Berlin Sundial Collaboration. *Ancient Sundials, Dialface ID 149, Paris, Inv. Nr. 2820*. Ed. by G. Graßhoff and E. Rinner. Berlin: Edition Topoi, 2014. DOI: 10.17171/1-1-1789.

Berlin Sundial Collaboration 2014b

Berlin Sundial Collaboration. *Ancient Sundials, Dialface ID 150, Paris, Inv. Nr. 2820*. Ed. by G. Graßhoff and E. Rinner. Berlin: Edition Topoi, 2014. DOI: 10.17171/1-1-1858.

Berlin Sundial Collaboration 2015a

Berlin Sundial Collaboration. *3D-Model of Dialface ID 477, Villa San Marco, Castellammare di Stabia, Italy, Inv. Nr. 62405*. Ed. by G. Graßhoff and E. Rinner. Ancient Sundials. Berlin: Edition Topoi, 2015. DOI: 10.17171/1-1-3978.

Berlin Sundial Collaboration 2015b

Berlin Sundial Collaboration. *3D-Model of Dialface ID 503, Delos, Inv. Nr. B4669*. Ed. by G. Graßhoff and E. Rinner. Ancient Sundials. Berlin: Edition Topoi, 2015. DOI: 10.17171/1-1-4094.

Berlin Sundial Collaboration 2015c

Berlin Sundial Collaboration. *Image of Dialface ID 124, Delos, Inv. Nr. B3652 (11023)*. Ed. by G. Graßhoff and E. Rinner. Ancient Sundials. Berlin: Edition Topoi, 2015. DOI: 10.17171/1-1-1264.

Berlin Sundial Collaboration 2015d

Berlin Sundial Collaboration. *Image of Dialface ID 124, Delos, Inv. Nr. B3652 (11023)*. Ed. by G. Graßhoff and E. Rinner. Ancient Sundials. Berlin: Edition Topoi, 2015. DOI: 10.17171/1-1-1266.

Berlin Sundial Collaboration 2015e

Berlin Sundial Collaboration. *Image of Dialface ID 477, Villa San Marco, Castellammare di Stabia, Italy, Inv. Nr. 62405*. Ed. by G. Graßhoff and E. Rinner. Ancient Sundials. Berlin: Edition Topoi, 2015. DOI: 10.17171/1-1-5534.

Fried and Unguru 2001

Michael N. Fried and Sabetai Unguru. *Apollonius of Perga's Conica: Text, Context, Subtext*. Mnemosyne Supplements 222. Leiden, Boston, and Köln: Brill, 2001.

Gibbs 1976

Sharon L. Gibbs. *Greek and Roman Sundials*. Yale Studies in the History of Science and Medicine 11. New Haven, CO and London: Yale University Press, 1976.

Heath 1921

Thomas Heath. *A History of Greek Mathematics, Vol. 1: From Thales to Euclid*. Oxford: Clarendon Press, 1921. URL: <https://archive.org/details/cu31924008704219> (visited on 17/7/2017).

Heiberg 1891–1893

Johann Ludwig Heiberg, ed. *Apollonii Pergaei quae Graece exstant cum commentariis antiquis*. 2 vols. Leipzig: Teubner, 1891–1893.

Heiberg 1910–1915

Johann Ludwig Heiberg, ed. *Opera Omnia: Cum commentariis Eutocii*. By Archimedes. 3 vols. Leipzig: Teubner, 1910–1915.

Kroeper and Krzyzaniak 1998

Karla Kroeper and Lech Krzyzaniak. "The Amun Temple Complex: Preliminary Report, Seasons 1 and 2". In *Naga Projet (Sudan): Egyptian Museum Berlin*. Ed. by F. Geus. Archeologie du Nil Moyen: Fouilles au Soudan et en Nubie, Vol. 8. Association pour la Promotion de l'Archeologie Nilotique, 1998, 203–207.

Neugebauer 1948

Otto E. Neugebauer. "The Astronomical Origin of the Theory of Conic Sections". *Proceedings of the American Philosophical Society* 92.3 (1948), 136–138. URL: <http://www.jstor.org/stable/3143581> (visited on 17/7/2017).

Neugebauer 1975

Otto E. Neugebauer. *A History of Ancient Mathematical Astronomy*. [HAMA.] 3 vols. Studies in the History of Mathematics and Physical Sciences 1. New York, Berlin, and Heidelberg: Springer Science, 1975. DOI: 10.1007/978-3-642-61910-6.

Schaldach 2006

Karlheinz Schaldach. *Die antiken Sonnenuhren Griechenlands: Festland und Peloponnes*. Frankfurt am Main: Harri Deutsch, 2006.

Stückelberger and Graßhoff 2006

Alfred Stückelberger and Gerd Graßhoff, eds. *Klaudios Ptolemaios. Handbuch der Geographie: Textband [2 Halbbände]*. In collab. with Florian Mittenhuber et al. Basel: Schwabe-Verlag, 2006.

Taliaferro and Fried 2013

R. Catesby Taliaferro and Michael N. Fried, trans. *Conics, Book I–IV: The Entire Surviving Greek Text in the Only English Translation*. By Apollonius of Perga. Green Lion Press, 2013.

Toomer 1990

Gerard J. Toomer, trans. *Conics, Book V–VII: The Arabic Translation of the Lost Greek Original in the Version of the Banū Mūsā*. By Apollonius of Perga. 2 vols. New York, Berlin, and Heidelberg: Springer Science, 1990. DOI: 10.1007/978-1-4613-8985-9.

TrigonArt, Bauer Praus GbR 2015 [2013]

TrigonArt, Bauer Praus GbR. *3D-Model of Dialfaces ID 190/625, Naga, Sudan*. Ed. by G. Graßhoff and E. Rinner. Ancient Sundials. Berlin: Edition Topoi, 2015 [2013]. DOI: 10.17171/1-1-2422.

Illustration credits

1 Berlin Sundial Collaboration 2015c (left) and Berlin Sundial Collaboration 2015d (right).

2 Diagram by Elisabeth Rinner, information resulting from Gibbs 1976, Schaldach 2006, and others.
3–4 Elisabeth Rinner.

5 Left: Elisabeth Rinner, based on the 3D model of the object (Berlin Sundial Collaboration 2015a).

Right: Photo from Berlin Sundial Collaboration 2015e.

6 Elisabeth Rinner, based on the 3D model of the object (TrigonArt, Bauer Praus GbR 2015 [2013]).

7–8 Elisabeth Rinner.

9–10 Elisabeth Rinner, based on the 3D model of the object (Berlin Sundial Collaboration 2015b).

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