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Dating by Month-Lengths Revisited

Summary

The chronological implications of the month-length evidence are re-examined on the basis of additional data, and newer astronomical theories and insights about the clock-time correction. The month-length evidence available by 2013 is internally consistent, and it confirms the former conclusions of 1982, although with slightly lowered confidence. It favors the High and disfavors the Middle chronologies with confidence levels between 95% and 99%. A Bayesian argument intimates that the High chronology (Ammiṣaduqa year 1 = 1702 BC) is roughly 25 times more probable than each of the other three main chronologies (1646, 1638, or 1582 BC). Independently, also the Ur III evidence points toward a High chronology (Amar-Sin year 1 = 2094 BC).

Keywords: Near Eastern Chronology; month-length dating; Venus Tablet; Ammiṣaduqa intercalations; clock-time correction.

Die chronologischen Implikationen der Belege für Monatslängen werden in diesem Beitrag anhand von zusätzlichem Material sowie von neueren astronomischen Theorien und Erkenntnissen über die Zeitkorrektur nachgeprüft. Die 2013 zur Verfügung stehenden Belege für Monatslängen sind in sich stimmig und bestätigen die Schlussfolgerungen von 1982, wenn auch mit etwas niedrigerer Konfidenz. Bei einem Konfidenzniveau zwischen 95 % und 99 % wird die lange Chronologie zu Ungunsten der mittleren Chronologien favorisiert. Ein Bayessches Argument verdeutlicht, dass die lange Chronologie (Ammiṣaduqa Jahr 1 = 1702 v. Chr.) ungefähr 25-mal wahrscheinlicher ist als jede der anderen drei Chronologien (1646, 1638 oder 1582 v. Chr.). Davon unabhängig deuten auch die Ur-III-zeitlichen Belege auf die lange Chronologie hin (Amar-Sin Jahr 1 = 2094 v. Chr.)

Keywords: Chronologie Altvorderasiens; Monatslängen; Venus-Tafel; Ammiṣaduqa-Interkalation; Zeitkorrektur.

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The suggestion to explore small prior probabilities for the additional intercalation (Section 5.2.3) is due to Jane Galbraith. Of course, all opinions and errors are my own.

1 Introduction

Month-length dating forms part of a tangled tale concerned with fixing the absolute chronology of the ancient Near East. This tale is based on evidence from history (back-reckoning using king lists, eponym lists, synchronisms, ...), archaeology (stratigraphy, pottery, ...), and natural science (C14-dating, dendro-chronology, volcanic activity, ...), including astronomy (Venus Tablet, solar and lunar eclipses, month-lengths). The internal relative chronology of the period in question, which ranges from the late third to the mid-second millennium, that is from the beginning of the Third Dynasty of Ur to the end of the First Dynasty of Babylon, is now agreed upon to within very few years, but its absolute position still is in doubt, and the disputes shift it forth and back over roughly 150 years. While the present paper concentrates on month-length dating, by necessity it must touch on some of the other parts also.[1]

The last comprehensive treatment of the month-length evidence has been that by Huber et al. in *Astronomical Dating of Babylon I and Ur III* (published in 1982),[2] followed by Huber's somewhat cursory re-takes and updates, spreading from 1987 to 2012.[3] These papers had reached the conclusion that the month-length evidence overwhelmingly favored the High Venus chronology (HC, Ammiṣaduqa year $1 = 1702$ BC).

The current re-examination has been triggered by the recent flurry of activity concerning the Old Assyrian eponym lists and the dendro-chronological dating of the Kültepe site. This activity has collected strong, and as it seems, equally overwhelming evidence in favor of the so-called Middle Venus chronologies. Barjamovic, Hertel, and Larsen, in a comprehensive monograph published in 2012, have settled on the traditional Middle Chronology (MC, Ammiṣaduqa year $1 = 1646$ BC).[4] De Jong (in a paper published in 2013) and Nahm (in a paper published in 2014) argue in favor of the Low Middle Chronology (LMC, Ammiṣaduqa year $1 = 1638$ BC).[5] Roaf (in a paper published in 2012) favors the Middle chronologies but advises caution.[6]

1 Note: The paper was written in early 2014 and is based on materials available by 2013.
2 Huber, Sachs, et al. 1982.
3 Huber 1987; Huber 1999/2000; Huber 2000; Huber 2012.
4 Barjamovic, Hertel, and Larsen 2012.
5 De Jong 2013; Nahm 2014.
6 Roaf 2012.

In view of this seemingly irreconcilable conflict it is worthwhile – and perhaps even mandatory – to re-examine the month-length evidence with the help of the currently available material: (i) a moderately increased data base, (ii) more modern astronomical theories, and (iii) a better insight into the clock-time correction. I shall concentrate on the methodological aspects, in order to check and possibly identify weak spots of the arguments.

New OB material has been supplied by Seth Richardson, new Drehem material by Robert Whiting, and I am offering heartfelt thanks to both. With regard to methodological aspects it is relevant to note that (i) it does not suffice to scan the electronic text catalogs for intercalations and day-30 dates – it is absolutely necessary to examine the cuneiform sources in detail and to rely on the judgment of specialists, and (ii) that more data do not necessarily imply improved chronological discrimination.

The Babylonian months are based on a lunar calendar, and their length alternates irregularly between 29 and 30 days. The Babylonian day began at sunset, and the Babylonian month began with the first visibility of the lunar crescent in the evening. According to Babylonian custom, immediately after sunset of day 29, day 30 would begin in any case. But if the moon became visible shortly thereafter, that is some 20–30 minutes after sunset, the day would be denoted 'returned' (Akkadian *turru*) to become day 1 of the following month (that is, the date would be changed *retroactively*, with the retroaction spanning some 30 minutes). The preceding month thus would become hollow (29 days). Otherwise the day would be 'confirmed' (*kunnu*) or 'rendered complete' (*šullumu*); see the Akkadian dictionaries for these verbs, and in particular Neugebauer's translation and commentary of ACT No. 200 Sect. 15 for the technical use of the terms in mathematical astronomy,[7] and the letter BM 61719 (CT 22, No. 167), where the writer asks for speedy information whether the day is *kunnu* or *turru*.

There are no intervals of 28 days between two calculated crescent sightings, and only rare intervals of 31 days (about once in a century, and therefore statistically irrelevant). It seems that a Babylonian day 30 always was followed by day 1, whether or not the crescent was sighted. Since the synodic month has 29.53 days, one expects that 53% of the months have 30 days and 47% 29 days. For randomly selected (wrong) chronologies we therefore expect an agreement rate of 53% between calculated and observed 30-day months, and 47% for 29-day months. These rates for wrong chronologies are based solely on astronomical theory. For a correct chronology the evidence from Neo-Babylonian administrative texts (mostly texts dated on day 30) gives an agreement rate with modern calculation of 67% (103 of 153 attestations). Actually, I find it more convenient to work with expected miss rates; for 30-day months these are 47% for a wrong, 33% for a correct chronology. If the data set contains also attestations of a few 29-day months, the miss rate for random wrong chronologies must be minimally adjusted upward.

7 O. E. Neugebauer 1955, 206.

The figure of 33% applicable to a correct chronology is an empirical estimate and as such is affected by a standard estimation error of about 3.8 percentage points. Apart from this estimation error we do not know for certain whether the NB miss rate is applicable also to OB and Ur III times. The data sets are not exactly comparable; NB and OB evidence mostly is from texts dated on day 30, while a substantial fraction of the Ur III evidence also derives from other, and possibly more reliable data (e.g. from regular deliveries: one sheep per day for the dogs of Gula).

The principal criticism voiced against month-length dating seems to be that it has not been proved that the Neo-Babylonian 33% rate for correct chronologies is applicable to Old-Babylonian and Ur III data. This criticism is beside the point. The central argument showing that a certain chronology is right (thereby simultaneously establishing that its competitors are wrong) consists in showing that the miss count of that chronology is significantly below that to be expected from a wrong chronology. This argument relies only on the theoretically secure rate of 47%. If the miss rate is not significantly below 47%, we simply shall be unable to reach a conclusion. The 33% rate is used only in an ancillary fashion, namely to add evidence that a certain chronology is wrong. I hope to clarify these issues in the discussion of the Ammiṣaduqa-Ammiditana data.

The Venus Tablet remains a central part of the evidence.[8] The paper by Nahm (published in 2014) contains a most recent, comprehensive discussion.[9] In view of the agreement of the pattern of intercalations with that of contemporary Old Babylonian texts we now know for sure that the first 17 years of the Venus Tablet correspond to the first 17 years of the Old Babylonian king Ammiṣaduqa (see Section 10.1 in the Appendix of this article). Moreover, we now know that we have the complete pattern of intercalations for those 17 years (more precisely: we know all intercalations contained in the interval from year 1 month VII to year 18 month VI), and in particular we know the exact distances between the months of that interval. Note that we do not know for sure whether year 1 is normal or whether it contains a second Ulūlu (VI_2). This uncertainty is of some relevance in connection with the Ammiditana date (see Section 5.2).

I believe that only the four main Venus chronologies (Ammiṣaduqa year 1 = –1701, –1645, –1637, –1581) have a realistic chance of being correct.[10] We distinguish them as High (HC), (High) Middle (MC), Low Middle (LMC), and Low Chronology (LC). This assertion in part is based on the Venus Tablet evidence and in part on the historical time window now considered to be feasible. Among the other chronologies that have been entered into the discussion in recent years, the Gasche-Gurzadyan chronology (year 1 = –1549) is incompatible with the lunar calendar, and the Mebert chronology (year 1 =

8 Reiner and Pingree 1975.

9 Nahm 2014.

10 I am using the astronomical year count, which differs by one year from the historical count – in the latter, the year 1 BC is followed by AD 1.

–1573) relies on some demonstrably wrong assumptions about the arcus visionis values and on some questionable textual emendations.[11]

Current evidence centered on dendro-chronology and Assyrian eponym lists points toward the Low Middle chronology (year 1 = –1637). On the other hand, for the middle two chronologies the Venus phenomena show statistically significant deviations of ±2 days against calculation, on a 1% significance level. In particular, for the Low Middle chronology and all four events the observations on average are about 2 days later than calculated, while for the High Middle chronology they are correspondingly earlier.[12] This holds if the Old Babylonian observing and recording practices were basically the same as the Late Babylonian ones. We do not know for sure how the Babylonian astronomers dealt with adverse weather conditions. Ordinarily, the LB observers inserted educated guesses for absent observations, possibly based on observations made one Venus period (8 years) earlier, with the remark 'not observed' (NU PAP). The OB observers might have used a more naïve approach, possibly causing a systematic shift. Werner Nahm hypothetically suggests that they might have written down the first date on which they could confirm that Venus had entered a new phase, either visibility or invisibility.[13] If so, bad weather would delay the observed phenomena, but his suggestion does not convince me. An even more simple-minded approach based on *actually observed* first and last visibilities seems to me at least as plausible. With this approach, bad weather would have symmetric effects, on average mutually canceling each other: it would not only delay first visibility, but also lead to an earlier begin of invisibility. I shall keep all four main chronologies as possibilities, since – as always with delicate data analytic arguments – there is a non-negligible residual risk of error. But in my opinion it is small enough to cast serious doubts on the middle chronologies.

2 Calculation of crescents

Theoretical crescent visibility shall be determined according to a recipe described by P. V. Neugebauer.[14] The position of the moon is calculated at the time of sunset or sunrise (more precisely: when the center of the sun is in the mathematical horizon), ignoring parallax and refraction. For these calculations I used the programs by Chapront-Touzé and Chapront (published in 1991),[15] but with improved values for the clock-time correction ΔT and the lunar orbital acceleration.

11 Gasche et al. 1998; Mebert 2010; Huber 2000; Huber 2011.

12 See the row with the medians in Tab. 2.2 of Huber 2000.

13 Nahm 2014.

14 P. V. Neugebauer 1929.

15 Chapront-Touzé and Chapront 1991.

The lunar crescent then is supposed to be visible shortly after sunset if the altitude h_{moon} of the moon at sunset exceeds a certain value h (the thin lunar crescent is not visible at the moment of sunset or sunrise itself). The critical value of h depends on the difference Δ in azimuth between sun and moon and h has been determined empirically. Thus the crescent is assumed to become visible on the first evening for which the altitude difference

$$\Delta h = h_{\mathrm{moon}} - h$$

is greater or equal zero (and to become invisible on the first morning for which this difference is less or equal zero). The tables for the critical value h given by P. V. Neugebauer,[16] and shortly before by Langdon, Fotheringham, and Schoch,[17] differ slightly. Both tables go back to Carl Schoch. Identifying the tables by the initials of the authors, I am following PVN, while Parker and Dubberstein[18] followed the earlier LFS version. See Tab. 1 and Fig. 1.

This method for calculating crescent visibility admittedly is dated. Its advantage is that it has been extensively tested against antique data (see the next section). There are more modern approaches by Schaefer and others, but in the absence of testing it is not known how well they perform with regard to observations made before the industrial revolution. Of course, the critical altitude h is not meant as a sharp limit, and the following section gives empirical evidence for the size of its uncertainty range.

In 1982 I calculated all 33 000 lunar crescents for Babylon (44.5 E and 32.5 N) between the years –2456 and +212. The following statistics may be of some interest. I am quoting the results of 1982; more modern programs and different choices of the clock-time correction ΔT cause only negligible minor variations. There were no 28-day months at all, but there were 20 months with 31 days. We believe that the Babylonian months never exceeded 30 days (even if the crescent did not appear), and therefore, a 31st day should be carried over to the next month. After carrying over the additional days of the 31-day months, there were 15 491 29-day months (46.9%) and 17 509 30-day months (53.1%).

Note that the difference in longitude between sun and moon on average changes by 12° in 24 hours. However, first visibility of the crescent depends not only on the difference in longitude, but also on lunar latitude. On the day before theoretical first visibility, Δh can be as low as $-14.3°$, and on the day of first visibility, it can be as high as 14.4°. Between these two days, the value of Δh increases by at least 6.2° and by at most 14.5°.

16 P. V. Neugebauer 1929, Tab. E 21.

17 Langdon, Fotheringham, and Schoch 1928, Tab. K.

18 Parker and Dubberstein 1956.

The 29- and 30-day months follow each other in a quite irregular and not easily predictable sequence, but which is not really random (i.e. there are discernible differences between this sequence and one obtained by tossing a biased coin). I checked it for

$\|\Delta\|$	h	
	PVN	LFS
0	10.4°	10.7°
1	10.4	10.7
2	10.3	10.6
3	10.2	10.5
4	10.1	10.4
5	10.0	10.3
6	9.8	10.1
7	9.7	10.0
8	9.5	9.8
9	9.4	9.6
10	9.3	9.4
11	9.1	9.1
12	8.9	8.8
13	8.6	8.4
14	8.3	8.0
15	8.0	7.6
16	7.7	7.3
17	7.4	7.0
18	7.0	6.7
19	6.6	6.3
20	6.2	
21	5.7	
22	5.2	
23	4.8	

Tab. 1 Critical altitudes h for crescent visibility, in dependence of the azimuth difference $|\Delta|$. The values are those of P. V. Neugebauer (PVN), and of Langdon, Fotheringham, and Schoch (LFS), respectively.

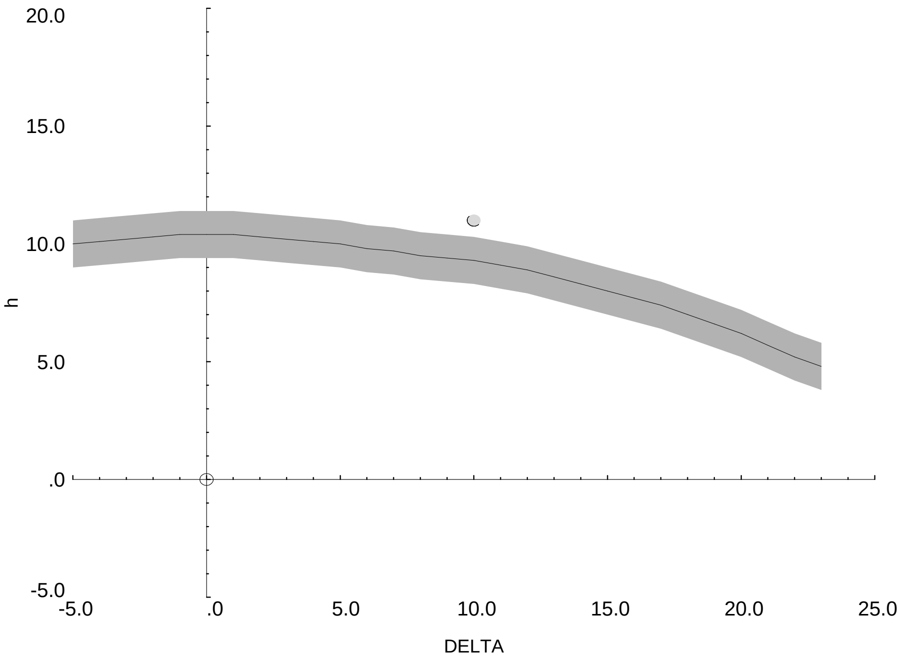

Fig. 1 Crescent visibility. Shown are the theoretical visibility curve of Tab. 1, PVN (solid), the gray zone (±1°), the sun (at the coordinate origin), and the thin lunar crescent (with the earth light). The figure is to scale.

periodicities by comparing month-lengths spaced up to 3000 months apart. The most pronounced period is 669 months or 54 years: month-lengths spaced 669 months apart agree in 81% of the cases. Note that 669 synodic months, or 54 years, is a well-known eclipse period (the so-called exeligmos). In particular, there are fewer and shorter runs (sequences of consecutive months of equal length) than in a truly random sequence. I found 410 runs of three consecutive 29-day months, and 100 runs of five consecutive 30-day months; longer runs did not occur.

3 The Late Babylonian evidence: astronomical texts

In preparation to the publication of *Astronomical Dating of Babylon I and Ur III* in 1982,[19] I had collected 602 lunar crescents in Late Babylonian observational astronomical texts. These are observations of the crescent and as a rule are accompanied by a measured time interval between sunset and moonset. Most of the texts are dated between 500 and

19 Huber, Sachs, et al. 1982.

150 BC. I had excluded calculated crescent data, that is crescents explicitly designated as 'not observed' (NU PAP), therefore the agreement of that data base with modern calculation may be better than the agreement to be expected from genuine observations. On the other hand, one should be aware that the ancient astronomers occasionally may have substituted educated guesses (based on observations shortly before or after the critical evening) or predictions when observational conditions were poor, without always stating the fact.

In the time when those astronomical texts were written the Babylonians had fairly accurate prediction methods. Between 641 and 591 BC they had developed methods for predicting the so-called Lunar Six (time differences between the rising and setting of sun and moon, near new and full moons).[20] Their methods for predicting the Lunar Six and the beginning of the month were based on observations made one Saros cycle, or 18 years, earlier; they have been elucidated by Brack-Bernsen.[21]

This observational material then was compared with modern calculations based on the PVN values of Tab. 1; it was not deemed extensive enough to model seasonal dependencies. Among the 602 crescents, there are 34, or 5.6%, discrepancies between observation and calculation. Of those, 30 correspond to marginal visibility conditions with $|\Delta h| < 1.0°$, that is to cases where the altitude of the moon was within $\pm 1°$ of the theoretical curve deciding visibility, see Fig. 1. Among the remaining four observations, one is a clear gross error, and two come from the same, poorly preserved tablet. This residual error rate is remarkably small. Note that according to modern experience, when data are recorded by hand, in the absence of proof reading gross error rates in the range between 1% and 10% are quite common.[22] I therefore assume that there was careful proof reading. Given the low residual error rate, observations with $|\Delta h| \geqslant 1.0°$, rather than being genuine observations that are less accurate than usual, just as likely either are gross scribal errors, or evidence for wrong modern dating of the tablet.

If we disregard gross errors, we thus have 598 observations, among which 30, or 5.0%, disagree with modern calculation. This disagreement rate is a statistical estimate and as such, assuming an underlying binomial distribution, is affected by a standard error of 0.9% percentage points. It is advisable to keep this statistical uncertainty in mind – with a similar but independent data set we might just as well have obtained a miss rate near 4% or 6% – but for the subsequent order-of-magnitude calculations we shall operate with 5%. Since a month-length depends on two crescents, the 5% miss rate for crescents translates into an approximate miss rate of 10% for month-lengths.

The following Tab. 2 gives the empirical distribution of sighted and not sighted crescents with calculated $|\Delta h| < 1.0$.

20 Huber and Steele 2007.

21 Brack-Bernsen 2011.

22 See Hampel et al. 1986, 25–28.

	not sighted	Δh	sighted	
	×××××××××	–0.9		
	×××××	–0.8		
	×××	–0.7	×××	
	×××	–0.6	××	
44	×××××××	–0.5	×	15
	××××××	–0.4	×××	
	××××	–0.3		
	×××	–0.2	×	
	××××	–0.1	××××	
		–0.0	×	
		0.0	××	
	×	0.1	×××××××	
	×	0.2	×	
	×	0.3	×××	
15	××	0.4	××××××	38
	×××	0.5	×××××	
	×××	0.6	××	
	×	0.7	×××	
	××	0.8	××	
	×	0.9	×××××××	

Tab. 2 Sighted and not sighted crescents in the Late Babylonian observational texts with the calculated value of $|\Delta h| < 1.0$ (from Huber, Sachs, et al. 1982, 27).

Based on this table I had tentatively proposed a probability model that disregarded gross errors but otherwise represented the observational astronomical data fairly well, namely:

- if $\Delta h < -1$, the crescent is never seen;
- if $-1 \leqslant \Delta h \leqslant 1$, the crescent is seen with probability $\frac{1+\Delta h}{2}$;
- if $\Delta h > 1$, the crescent is always seen.

Thus, near $\Delta h = 0$ the chance of seeing the crescent is roughly 50%, and for $\Delta h = -0.8$ the probability of sighting the crescent drops to 10%. By averaging over the intervals we obtain that for $-1 \leqslant \Delta h \leqslant 0$ the crescent is not sighted with probability 0.75 and sighted with probability 0.25, while for $0 \leqslant \Delta h \leqslant 1$ it is not sighted with probability 0.25 and sighted with probability 0.75. These theoretical $3:1$ ratios are close to the empirical ratios $44:15$ and $15:38$ of Tab. 2.

Note that for genuine observations the situation is not symmetric: if a text claims that the crescent had been seen, but calculation gives a negative $\Delta h \leqslant -1.0°$, we are practically guaranteed to have a gross scribal error or a wrong date. But if the crescent had not been seen with $\Delta h \geqslant 1.0°$, it is possible that the sighting had failed because of poor atmospheric conditions. However, this asymmetry does not manifest itself in the Late Babylonian data and therefore was not modeled.

On the basis of a long sequence of 33 000 calculated crescents (with Δh values rounded to the nearest multiple of $0.1°$), the above probability model yields that the crescents would be observed one day early or late in 2.3% of the cases, respectively, resulting in a calculated miss rate of 4.6%. This is well within the statistical uncertainty of the observed miss rate, but for the model calculations of Sections 7 and 8, I preferred to increase the width $\pm d$ of the gray zone from $\pm 1.0°$ to $\pm 1.1°$, in order to obtain a miss rate of 5.1%, closer to the observed value.

4 The Neo-Babylonian evidence: administrative texts

Non-astronomical texts – mostly administrative texts from between 650 and 450 BC, dated on day 30, where such a date would appear to imply that the month had 30 days – have a substantially higher disagreement rate against calculation. In 1982 we found 153 suitable texts, with a disagreement rate of $^{50}/_{153} = 32.7\%$ for month-lengths.[23] This translates into about 17% with regard to crescents, and there are about 8% cases with $|\Delta h| \geqslant 1.0°$.[24] It is difficult to separate the causes of these discrepancies into careless dating, less reliable observations made by non-astronomers, and gross scribal errors. I now repeated the calculations with newer programs and the best currently available ΔT-values for the Neo- and Late-Babylonian period.[25] The results were practically identical.

The correct chronology with 50 misses does not give the best possible fit. Among 20 000 alignments of the 153 observed month-lengths along a calculated sequence there were 16, or 0.08% alignments with 50 or fewer misses. The best fit had 46 misses, and fits with 50 and 49 misses were found 669 months, or 54 years, before and after the true date, respectively (remember the 669 months lunar period!). This means that a randomly chosen (wrong) alignment has a chance of 0.08% of hitting an equally good or better agreement than the true one. And there are good chances to find an equally good fit exactly 669 months or 54 years before or after the true one.

23 Huber, Sachs, et al. 1982, 28–29.

24 Huber, Sachs, et al. 1982, 28–29.

25 The ST82f formula for ΔT of Huber and De Meis 2004, 25.

Reassuringly, we can conclude that the agreement of the recorded 30-day months with the calculated 30-day months is significantly better for the true chronology than for a wrong chronology. However, even with 153 recorded month-lengths the agreement ordinarily is not good enough to permit independent dating in the absence of other evidence, that is, unless we can narrow down the candidate chronologies to a few precise years. A detailed quantitative discussion is required.

Theoretical arguments involving the miss rates of *wrong* alignments are based on astronomical theory, namely on the 29.53 days length of the synodic month and the resulting miss rate of 47% for randomly aligned 30-day months. The binomial distribution gives a good approximation to the distribution of empirically observed miss rates, see Fig. 2.

Arguments involving the miss rate of the *correct* chronology are more delicate, quite apart from the question whether the miss rates for NB and OB times were the same. For true alignments this miss rate also follows a binomial distribution, but with a lower value of p, see Fig. 7 of Section 8. For arguments relying on the miss rates for true alignments one should keep in mind that the disagreement rate of $p = 50/153 = 32.7\%$ between observed and calculated month-lengths is a mere estimate, and as such is affected by a standard error of $\sqrt{p(1-p)/n}$, or about 3.8 percentage points.

Since the observed miss count is a random quantity, some luck is involved. Let us fix the idea by arbitrarily assuming that the true disagreement rate is 32.7%, and that we are trying to find a date on the basis of an independent new sample of 153 month-lengths. Note that this is a much larger sample than we can hope for in the case of Ammiṣaduqa + Ammiditana. Then for a correct alignment the chances are 27% that the observed miss count is less or equal 46, and also 27% that it is greater or equal 54. With a correct alignment and good luck, we perhaps might have obtained 46 misses and a miss rate of 30.0%, with bad luck perhaps 54 misses and a miss rate of 35.3%. With a wrong chronology the expected number of misses is $0.47 \times 153 = 72$, and the binomial probability of obtaining 46 or fewer misses is 0.0000145, and of obtaining 54 or fewer misses is 0.00224.

Assume now that we desire to fix the true chronology with an error probability of 1%. Then the probability that the best of 690 random trials with wrong alignments achieves 46 or fewer misses is approximately 1% ($\approx 690 \times 0.0000145$). In such a lucky case, picking the correct date based solely on month-lengths is eminently feasible – in a line-up of 690 candidates we pick the true one with 99% chance. On the other hand, if we are unlucky, the probability that the best of 4 random trials with wrong alignments achieves 54 or fewer misses is about 1% ($\approx 4 \times 0.00224$), which would be just sufficient to pick the correct chronology in an Ammiṣaduqa-like case with 99% chance in a line-up of four precisely fixed candidate chronologies.

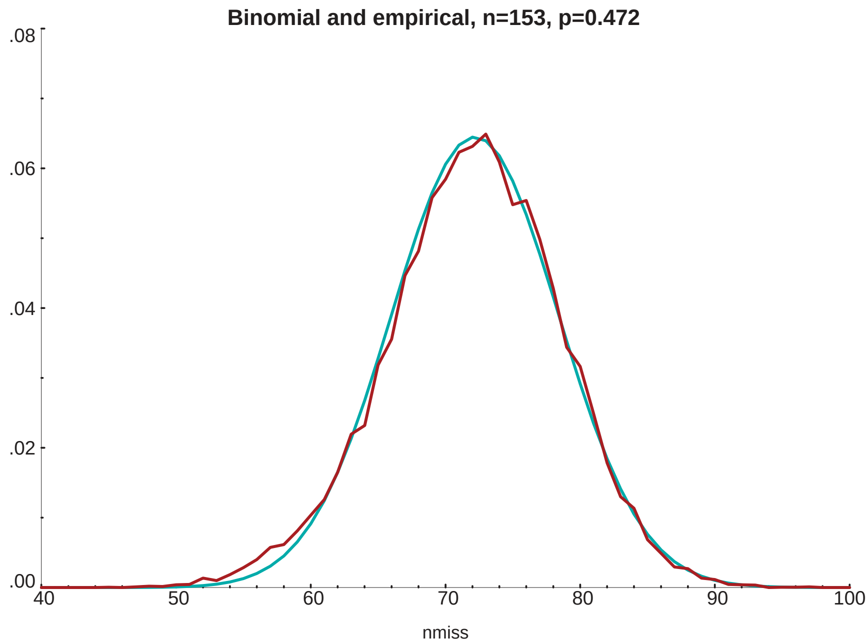

Fig. 2 Comparison between the binomial distribution ($n = 153,\ p = 0.472$, blue) and the empirical frequencies obtained from 20 000 alignments of the Late Babylonian data (red). (The LB data contain 149 30-day months and 4 29-day months, and the p of the binomial distribution was adjusted accordingly from 0.47 to 0.472.)

A detailed discussion of the NB material follows. The 153 texts contain 4 attested 29-day months, 3 of which agree with calculation, and one calculates as 30 days, possibly shortened by marginal calculated visibility ($\Delta h = 0.2°$) at the beginning of the month. Tab. 3 lists the results of a comparison of the remaining 49 months that calculate as having 29 days but where the texts have a day 30. This is an extract from *Astronomical Dating of Babylon I and Ur III*,[26] but re-calculated with newer programs.

We note that of those 49 months 14 have marginal visibilities $(-1.1 \leqslant \Delta h \leqslant 0)$ at the beginning of the month, and 14 have marginal visibilities at the end of the month $(0 \leqslant \Delta h \leqslant 1.1)$. The former may lengthen the observed month at the beginning, the latter at the end. One month has marginal conditions both at the beginning and the end. For the remaining 22 months the mismatch to calculation cannot be explained by marginal visibility; for them, we have $1.6 \leqslant \Delta h \leqslant 6.8$ at the end of the month. Incidentally, the big list of 33 000 calculated crescents shows that at the end of calculated 29-day months Δh ranges from 0 to 10.9.

26 Huber, Sachs, et al. 1982, 51–55.

Syzygy Number	begin of month		end of month	
	Δ*h* before 1st visibility	Δ*h* at 1st visibility	Δ*h* before 1st visibility	Δ*h* at 1st visibility
5537	−6.2	3.6	−9.1	.0
5925	−4.0	6.8	−10.9	.1
5827	−4.2	6.7	−11.2	.1
5600	−.8	7.7	−7.4	.2
5610	−5.6	4.7	−7.8	.2
5917	−7.0	5.7	−10.6	.3
5687	−3.9	6.0	−9.1	.3
5315	−6.1	3.6	−8.6	.4
5722	−6.5	6.1	−10.4	.7
5854	−3.8	7.2	−9.6	.8
11235	−1.4	9.4	−8.6	.9
9035	−3.8	6.3	−8.1	.9
5597	−7.0	4.9	−8.4	.9
6219	−3.4	5.5	−7.8	1.0
5683	−.6	10.3	−7.3	1.5
4726	−3.1	8.1	−10.0	**1.6**
5781	−2.9	9.5	−9.1	**1.7**
5363	−1.2	8.2	−5.4	**2.0**
5766	−5.1	8.1	−10.6	**2.0**
5045	−1.0	8.3	−7.0	2.2
5446	−4.3	7.9	−8.4	**2.2**
5748	−5.3	5.9	−8.1	**2.7**
5570	−5.2	8.5	−10.0	**2.8**
5774	−1.8	7.6	−6.6	**3.1**
5802	−4.3	8.1	−9.9	**3.4**
5530	−1.4	10.7	−8.2	**3.5**
5526	−4.3	7.0	−7.1	**3.6**

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Syzygy Number	begin of month Δh before 1st visibility	begin of month Δh at 1st visibility	end of month Δh before 1st visibility	end of month Δh at 1st visibility
5877	−4.7	9.0	−9.5	**3.6**
5678	−4.8	8.7	−9.5	**4.3**
5327	−.2	9.3	−4.3	4.4
6001	−4.9	9.3	−9.6	**4.4**
4996	−5.4	7.8	−9.1	**4.7**
5850	−2.2	10.5	−7.2	**5.0**
5334	−1.5	11.6	−7.3	**5.1**
5890	−4.4	9.6	−8.1	**5.3**
6377	−.0	12.7	−6.3	5.3
5405	−.6	11.6	−6.1	5.4
5329	−.0	10.3	−5.4	5.5
5499	−2.0	11.7	−6.7	**5.5**
5903	−4.0	9.9	−7.0	**6.1**
6194	−2.5	9.6	−5.3	**6.4**
5050	−1.3	11.8	−6.4	**6.8**
5654	−.2	13.3	−5.5	7.4
5863	−.8	13.0	−5.5	7.7
5899	−.2	12.1	−4.2	8.5
5008	−1.1	11.6	−4.5	8.7
5677	−.3	12.5	−4.8	8.7
6392	−.1	13.7	−3.5	9.0
5442	−.8	13.2	−4.8	9.5

Tab. 3 Neo-Babylonian data: 49 texts dated on day 30, whereas calculation indicates a 29-day month. The table is sorted according to the calculated Δh at 1st visibility at the end of the month. Where a mismatch cannot be explained as a gray-zone effect, the Δh value is shaded.

I am not sure how Tab. 3 is to be interpreted. Clearly, there is a crowding of values in the marginal visibility zones, both at the beginning and the end of the month. For the remaining 22 months, or 45% of the total, at the end of the month Δh is fairly evenly distributed over the range between 1.6 and 6.8, and thus the mismatch cannot simply be explained by expanding the marginal visibility zones. I believe the most plausible suggestion is that between 40% and 50% of the day-30 dates are 'overhang' dates, on which a scribe wrote day 30 instead of day 1 of the following month. In these cases the following day would be day 2 of the new month. I shall elaborate on this idea in Section 7.

5 On the discriminatory power of month-lengths

I shall concentrate on methodological aspects, but shall illustrate them by discussing in detail two concrete data collections that involve crucial aspects and difficulties: month-lengths (1) from the reign of Ammiṣaduqa, and (2) from the reign of Ammiditana. An early draft had contained also a detailed discussion of (3) the Hammurabi-Samsuiluna and (4) the Ur III evidence, both being less conclusive, but for reasons of space I now give only brief summaries. To avoid over-burdening the discussion, I shall relegate most technical details to Sections 7, 8, and 9 below, and to Section 10, the Appendix listing the data collections.

A perennial methodological problem is that our pool of month-length data may be too small to guarantee a decision. Even in the absence of grosser errors, such as erroneous intercalations, the unavoidable problem is the randomness of the miss counts. With some luck, the correct chronology may give a lower than expected miss count and force a decision. But if it accidentally gives a high miss count, the situation may remain undecided. With the miss counts of wrong chronologies opposite problems apply. More new data will not necessarily sharpen the decision – extreme counts will tend to regress toward the average (Galton's law of 'regression to mediocrity'). For example, in the case of Ammiṣaduqa to be discussed in Section 5.1, addition of 5 more month-lengths resulted in a poorer separation between the putative right and wrong alignments. In order to illustrate the intrinsic variability of small sample statistics – and to raise a warning signal against the temptation of over-interpretation – I shall present the analysis of the Ammiṣaduqa data both in terms of the smaller earlier and the increased later sets. Note that in critical cases elimination of a single mismatch by a minute change of ΔT can dramatically lower the P-values (minimum rejection levels), see Section 5.2.1.

Repeatedly, doubts have been raised whether the Old Babylonian month-lengths, i.e. month-lengths derived from texts dated on day 30, obey laws comparable to those of the Neo-Babylonian ones, in particular whether the NB miss rate of 33% is applicable. If it comes to the worst, the miss counts for the correct OB chronology might be no better than for wrong ones, and then the month-lengths would be useless for dating purposes. But at least in principle – that is, if the sample size is large enough – the month-length data can be used to settle this question in a methodological clean fashion, namely by testing whether one of the four main Venus chronologies gives a better fit than the best of four random wrong chronologies. Technically, this means that we should show by a statistical test that the best of the four miss counts is significantly better than the best of four random draws from a binomial distribution corresponding to wrong chronologies. If successful, this test (which is based only on the theoretically secure rate $p = 0.47$) would not only establish that one of the four chronologies is correct. It would also imply correctness of the singled-out best chronology and wrongness of the other three, and that the miss rate for a correct chronology – while not necessarily equal to the NB value – is well below that for a wrong chronology also for OB data.

But what sample size would we need? The Ammiṣaduqa sample sizes of 27 or 32 are not good enough. Assuming that the miss rate for correct chronologies is close to the NB value of 33%, I have estimated (with the help of some rough order-of-magnitude calculations with the binomial distribution) that one would need 60 or more month-lengths for such a test to have a fair chance of being successful. Actually, with some luck we shall squeeze by with a total of 49 data by combining the Ammiṣaduqa and Ammiditana samples in Section 5.2.1.

5.1 Case 1: Ammiṣaduqa

This subsection is concerned with the question whether and when the best fitting chronology can be declared being the correct chronology. It also illustrates that more data do not necessarily improve the discriminatory power.

In the case of the reign of Ammiṣaduqa we have four distinct, precisely fixed main chronological possibilities: Ammiṣaduqa year 1 = –1701, –1645, –1637, or –1581. For each of these four possibilities the syzygy numbers of the months from year 1 month VII to year 18 month VI are astronomically fixed by the Venus data. If Ammiṣaduqa year 1 is a normal year, they imply that month I of that year has the syzygy numbers –8666, –7974, –7875, or –7183, respectively.[27]

I first shall discuss the set of 27 day-30 dates available by 2010. Fig. 3 plots the binomial distributions corresponding to $p = 0.33$ on the left hand side (the miss rate corresponding to the Neo-Babylonian control material for a correct chronology) and to $p = 0.47$ on the right hand side (the theoretical rate for random wrong chronologies). There is considerable overlap between the two distributions. For a correct chronology we expect a miss count of 8.9, with a standard deviation of $\sqrt{n\,p\,(1-p)} = 2.4$. For a wrong chronology the expected count is 12.7, and for the best of four wrong chronologies the expected count is 10.0.

27 These numbers continue the syzygy count of Goldstine 1973 backward to the 2nd and 3rd millennium.

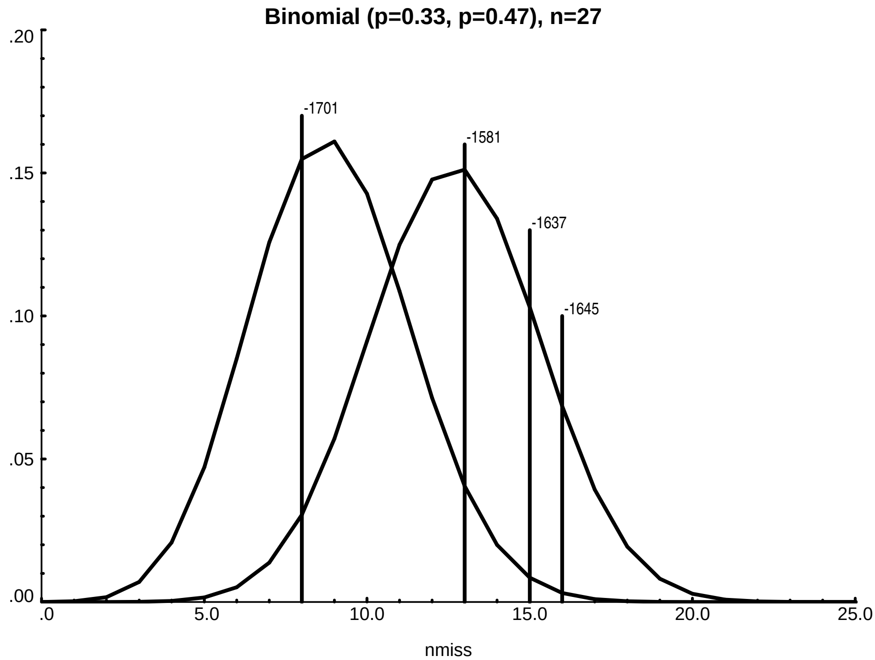

Fig. 3 Ammiṣaduqa data (set available in 2010). Binomial distributions for $p = 0.33$ and $p = 0.47$, $n = 27$. The vertical lines indicate the number of misses obtained for the 4 main chronologies with the Ammiṣaduqa data: –1701: 8, –1645: 16, –1637: 15, –1581: 13.

For the four main chronologies we obtain miss counts of 8, 16, 15, and 13 respectively, see Fig. 3. We expect that one of the four main chronologies is correct and is drawn from the left-hand distribution, while the other three are wrong and are drawn from the right-hand distribution. The figure clearly is consonant with this assumption; it suggests that –1701 is correct, and that the other three are unlikely in different degrees. Actually, the miss count for –1701 is below the expected value for a correct chronology by one unit, and the other three counts are all above the expected value for a wrong chronology.

While this data set clearly favors the –1701 chronology, the sample size is not large enough to force a decision in its favor. The probability that a random wrong chronology yields 8 or fewer misses is 0.052. But since we have picked the –1701 chronology not for extraneous reasons, but rather because it was the best of four, we should consider the probability that *the best of four random wrong chronologies* yields 8 or fewer misses; this probability is $1 - (1 - 0.052)^4 = 0.19$.

Through a Bayesian approach we can quantify the intuitive impression that –1701 is best and –1645 worst by assigning equal prior probabilities to the four chronologies. Then, their posterior probabilities are proportional to

$$\left(\frac{p\,(1-q)}{(1-p)\;q}\right)^{k}$$

where $p = 0.33$, $q = 0.47$, and k is the number of misses. For the 2010 data set they calculate as:

–1701: 0.927, –1645: 0.008, –1637: 0.015, –1581: 0.049.

Note that if all k are increased by the same constant, the posterior probabilities stay the same – this means that the Bayesian approach ignores the absolute quality of the four fits and is in particular unable to tell you whether or not all four are wrong. A disadvantage of all Bayesian approaches is that they have to rely on the Neo-Babylonian value of p.

By 2013, 5 more day-30 dates had become available. The result is depicted in Fig. 4. The –1701 chronology still is ahead, but its miss count of 12 now exceeds the expected value of 10.6 for a correct chronology by one unit, while the other three are at or above the expected value of 15.0 for a wrong chronology.

The 2013 Ammiṣaduqa data set is less able to assert correctness of the –1701 chronology than the 2010 set. While with the earlier set the lowest miss count was one unit below the expected value for a correct chronology, with the later set it is one unit above the expected value, and the probability that a random wrong chronology yields 12 or fewer misses is 0.18. The miss count of 12 of the High Chronology lies between the expected miss count for a correct chronology (10.6) and the count expected for the best of four wrong chronologies (12.2). All these number lie well within statistical variability; note that the standard deviation $\sqrt{n\,p\,(1-p)} = 2.7$ of the miss count for the correct chronology exceeds the difference between the last two numbers. (By the way, the standard deviation of the miss count of the best of four wrong chronologies is 2.0.)

The evidence does not suffice to establish correctness of the High chronology, but if the miss rate of 0.33 of the NB data is even approximately applicable, we can confidently (with better than 99% confidence) reject correctness of the traditional –1645 Middle chronology: the P-values are 0.45% for the 2010 set and 0.19% for the 2013 set.

For the 2013 set the posterior probabilities calculate as

–1701: 0.736, –1645: 0.012, –1637: 0.126, –1581: 0.126.

We summarize: the Ammiṣaduqa month-length evidence points in favor of the High chronology and disfavors the –1645 chronology.

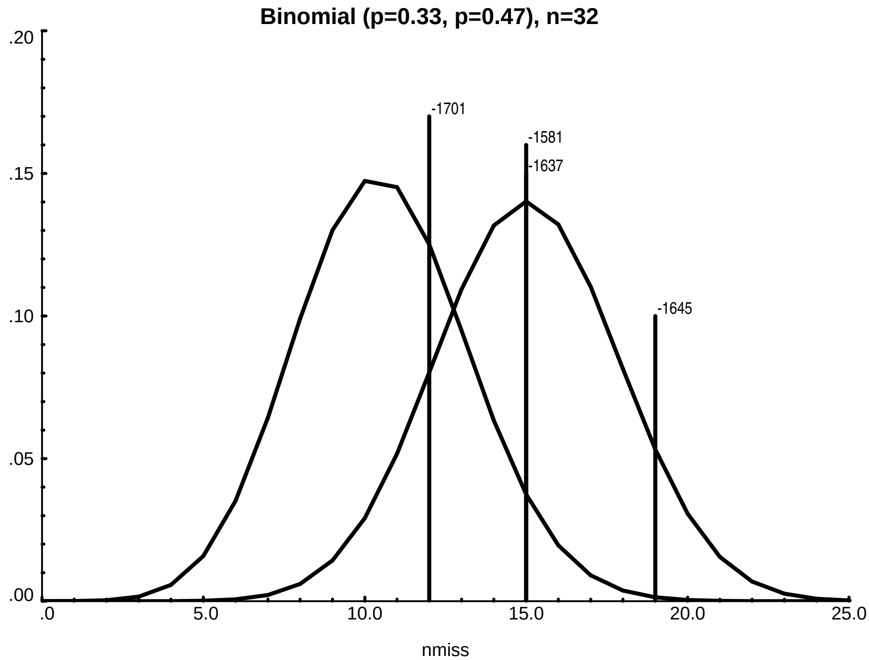

Fig. 4 Ammiṣaduqa data (set available in 2013). Binomial distributions for $p = 0.33$ and $p = 0.47$, $n = 32$. The vertical lines indicate the number of misses obtained for the 4 main chronologies with the Ammiṣaduqa data: −1701: 12, −1645: 19, −1637: 15, −1581: 15.

5.2 Case 2: Ammiditana

In the case of Ammiditana, the king of Babylon immediately before Ammiṣaduqa, we have 13 (set available since 1982), or 17 (set available in 2013) usable attestations of 30-day dates (from Ammiditana years 24 to 36). The problem here is that the positions of the intercalary months are not fixed by Venus observations as in the case of Ammiṣaduqa. Specifically, the question is whether the Ammiditana segment joins snugly in front of Ammiṣaduqa. Note that for the 7 years between Ammiditana 34 and Ammiṣaduqa 3 only a single intercalation is attested (see Section 10.2 in the Appendix of this article). So we should consider the possibility that there is an unattested intercalation near the boundary (for example a XII_2 in Ammiditana year 36, or a VI_2 in Ammiṣaduqa year 1; these two choices shift all currently attested Ammiditana month-lengths by one month, but do not interfere with their relative distances). I think it is advisable, if not mandatory, to take the uncertainty into account and to consider the possibility of an additional intercalation. In Tabs. 4–5, the results are identified by '+0' without, by '+1' with such

	Ammiṣaduqa year 1	Syz.no. of year 1	Aṣ	+0 Ad	+0 Ad+Aṣ	+1 Ad	+1 Ad+Aṣ	min Ad	min Ad+Aṣ
No. of months			27	13	40	13	40	13	40
High	−1701	−8666	8	8	16	2	10	2	10
High Middle	−1645	−7974	16	5	21	8	24	5	21
Low Middle	−1637	−7875	15	8	23	11	26	8	23
Low	−1581	−7183	13	9	22	5	18	5	18

Tab. 4 Counts of misses for Ammiṣaduqa and Ammiditana (sets available in 2010).

an additional intercalation, and 'min' gives the lower of the two counts. With '+1' the Ammiditana block as a whole is shifted one month.

When considered by themselves, the Ammiditana data lead to similar conclusions as the Ammiṣaduqa data: both favor the High chronology, see Tabs. 4–5, and compare Figs. 3–4 with Fig. 5. We may treat the two data sets as two independent witnesses. They are concordant, but not quite conclusive when taken separately. There are more promising approaches, namely by combining the two sets. I shall discuss three possible approaches.

Firstly, we may form a working hypothesis on the basis of one set and then test it on the basis of the other. Or secondly, we can pool the data, forget about the evidence of the components and proceed on the basis of the joined set. A third possibility is to combine the evidence from the different sets by Bayesian methods. To some extent the choice of method is a matter of taste. Personally, I think that in our case the first approach, testing a working hypothesis, is the cleanest (and clearest). Others might better like the third, Bayesian approach.

The approaches are complementary. Statistical tests can assess absolute quality, but have difficulties measuring relative merits, while with Bayesian approaches the opposite applies. In our particular case the first approach is suitable for establishing correctness of a chronology, the second for establishing wrongness of selected chronologies, and the third for assigning relative probabilities.

From now on we shall concentrate on the 2013 data set. The joined Ad + Aṣ data of Tab. 5 suggest that in the +0 column all four alignments are wrong: none of the counts is below the value 23.0 expected for a wrong chronology, and all exceed the value 16.2 expected for a correct chronology by more than twice their own standard

	Ammiṣaduqa year 1	Syz.no. of year 1	Aṣ	+0 Ad	+0 Ad+Aṣ	+1 Ad	+1 Ad+Aṣ	min Ad	min Ad+Aṣ
No. of months			32	17	49	17	49	17	49
High	–1701	–8666	12	11	23	3	15	3	15
High Middle	–1645	–7974	19	6	25	10	29	6	25
Low Middle	–1637	–7875	15	9	24	13	28	9	24
Low	–1581	–7183	15	10	25	7	22	7	22
Expected number of misses (± standard deviation)	correct		10.6 ±2.7	5.6 ±1.9	16.2 ±3.3	5.6 ±1.9	16.2 ±3.3		
	wrong		15.0 ±2.8	8.0 ±2.1	23.0 ±3.5	8.0 ±2.1	23.0 ±3.5		
	best of 4 wrong		12.2 ±2.0	5.9 ±1.4	19.4 ±2.4	5.9 ±1.4	19.4 ±2.4		

Tab. 5 Counts of misses for Ammiṣaduqa and Ammiditana (sets available in 2013).

deviation of 3.3. Thus, either a snug joining of the two data sets (+0) is wrong, or all four chronologies are wrong, or OB month-lengths are worthless for dating purposes.

On the other hand, in the +1 column the miss-counts match the assumption that we have one correct and three wrong chronologies. The count (15) for the HC turns out even better than what we would expect (16.2) for the correct chronology, and the counts for the Middle chronologies are devastatingly poor.

If indeed one of the four chronologies is correct, as is generally assumed, and if OB month-length data can provide valid evidence, a comparison between the +0 and +1 columns thus furnishes strong arguments in favor of an additional intercalation, and in favor of the High chronology, as well as against the Middle chronologies.

Fig. 5 is analogous to Figs. 3–4. It plots the binomial distributions corresponding to $p = 0.33$ (the miss rate corresponding to the Neo-Babylonian control material for a correct chronology) and to $p = 0.47$ (the theoretical rate for random wrong chronologies), and in addition it also shows the distribution of the counts for the better of two random draws from the right hand distribution. Correspondingly, the vertical lines indicate the better of the two Ammiditana counts (without and with the additional intercalation).

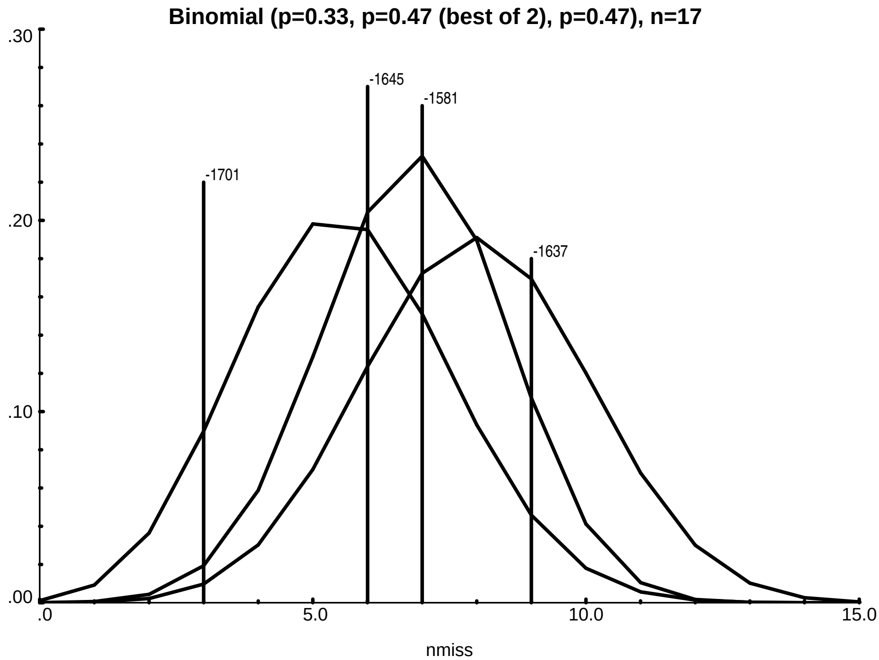

Fig. 5 Ammiditana data (set available in 2013), $n = 17$. Leftmost: binomial distribution for $p = 0.33$ (correct chronology); rightmost: for $p = 0.47$ (wrong chronology). In between the two, also the distribution of the best of two wrong chronologies is shown. The vertical lines indicate the number of misses obtained for the 4 main chronologies with the Ammiditana data. The lines correspond to the numbers obtained for the best of two fits (without or with the additional intercalation): −1701: 3, −1645: 6, −1637: 9, −1581: 7.

Back in 1982, for the −1701 chronology and the 13 month-lengths then available I had obtained a single miss when assuming an additional intercalation. For the same data the newer programs gave two misses. It turns out that the uncertainty of ΔT is such that for one of the Ammiditana month-lengths the decision between 29 and 30 days is ambiguous. The newer programs, which allow to vary ΔT, show not only that with the default ΔT the miss counts correspond to a local maximum for both Ammiṣaduqa and Ammiditana, but also that if ΔT is lowered by merely 3 minutes (that is, if c is changed from 32.50 to 32.35), the Ammiditana miss counts become 1 for the 1982/2010 set, 2 for the 2013 set. See Section 9 and Figs. 8, 9, 10, 11, and 12.

5.2.1 *Ammiṣaduqa results used as a working hypothesis*

The Ammiṣaduqa data – both the Figs. 3–4, and the Bayesian posterior probabilities – suggested that the HC is the correct chronology, but they did not suffice to establish it on the 5% level. We now use the HC as our working hypothesis, and we have to test the hypothesis that HC is wrong with the help of the Ammiditana data.

We take the 2013 month-length data and the miss counts of Tab. 5. If the HC is wrong, the Ammiditana miss count is distributed like a draw from a binomial distribution with $n = 17$ and the parameter $p = 0.47$, whether or not we assume the presence of an additional intercalation. The smaller of the two miss counts (without and with the additional intercalation) then is distributed like the smaller of two draws from this binomial. With the default ΔT, the smaller of the observed miss counts is 3 (see Tab. 5), and the probability of achieving $\leqslant 3$ misses is 2.4%. If we decrease ΔT by 3 minutes, the Ammiditana miss count for –1701 is decreased by 1 unit, and the minimum rejection level is reduced from 2.4% to 0.50%. Calculations with any of the other chronologies no longer are relevant. In other words, we reject wrongness of the HC on a level below 3%, possibly below 1%.

A possible criticism that might be raised against these calculations is that we draw pairs of chronologies spaced by one month, and so the draws are not quite random. But an empirical test (comparing pair-wise random draws with single draws from the calculated sequence of month-lengths) shows that the approximation nevertheless is excellent.

We emphasize that this test relies only on the secure rate of $p = 0.47$. In addition to confirming that one of the four chronologies is correct, namely the HC, it simultaneously implies that the Old-Babylonian miss rate for a correct chronology is substantially below 47%, and that the other three chronologies are wrong.

We summarize that we can confirm the HC with better than 95% confidence, and if we are willing to lower ΔT by 3 minutes against the arbitrarily assumed default formula, it is confirmed even with over 99% confidence.

5.2.2 *Joining the Ammiṣaduqa and Ammiditana data*

Alternatively, we may join the two data sets. The counts are shown in Tabs. 4–5, and the situation is depicted in Fig. 6. We again expect that one of the four main chronologies is correct and is drawn from the left-hand distribution ($p = 0.33$), while the other three are wrong and are drawn from the rightmost distribution ($p = 0.47$). Since for each chronology we again are considering the better matching of two possibilities, the vertical lines indicate the lower value of the two counts, and the distribution of the lesser of two independent draws from the right-hand binomial ($p = 0.47$) is depicted in the middle. There is considerable overlap between the distributions, but the figure clearly suggests that –1701 is the correct chronology and that both middle two chronologies are wrong. This can be quantified by calculating minimum rejection levels.

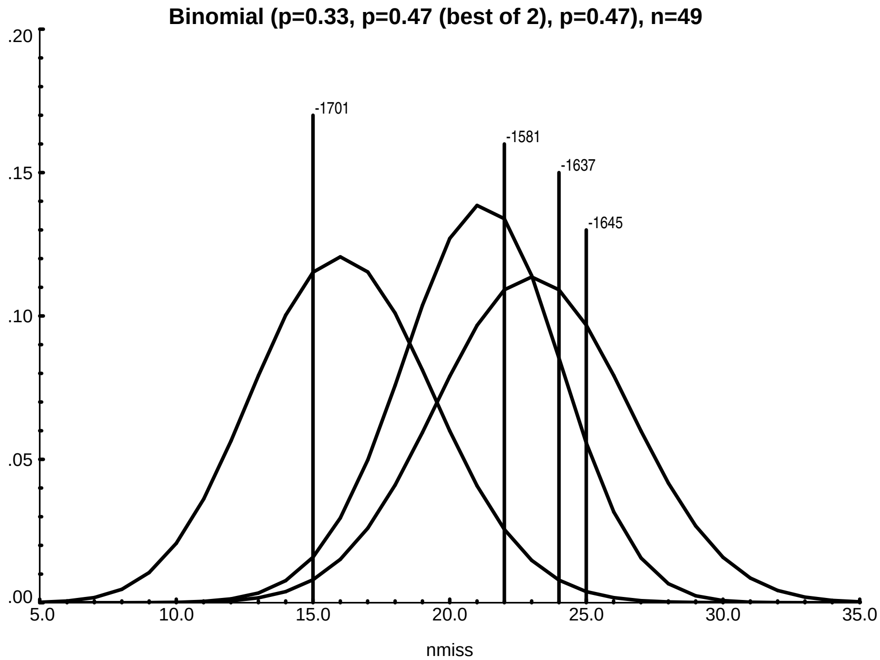

Fig. 6 Ammiditana + Ammiṣaduqa data (set available in 2013). Binomial distributions for $p = 0.33$ (correct chronology) and $p = 0.47$ (wrong chronology); $n = 49$. In between the two, also the distribution of the best of two wrong chronologies is shown. The vertical lines indicate the number of misses obtained for the 4 main chronologies with the combined Ammiṣaduqa-Ammiditana data. The lines correspond to the numbers obtained for the best of two fits (that is without or with the additional intercalation): −1701: 15, −1645: 25, −1637: 24, −1581: 22.

We perform statistical tests between the hypothesis H (chronology correct, $p = 0.33$) and the alternative A (chronology wrong, $p = 0.47$). Let x be the number of misses and assume that the binomial distributions $B(n, p)$, with the above values of p, and $n = 49$, give adequate approximations for the probability distribution of the miss counts.

(1) Test A against H

In this case, we have to test A (wrongness of the best fitting chronology). That is, we ought to check whether the best alignment we had found (among 4 chronologies and 2 intercalation patterns for each) fits significantly better than what can be expected from the best of 8 randomly chosen wrong alignments. We run into similar sample size problems as above with the Ammiṣaduqa data, but the rejection level (the probability that *the best of four random wrong chronologies* yields 8 or fewer misses) is somewhat reduced, namely to 11.1% for the 2013 set. This number is conservative: the (somewhat arbitrary default) formula for ΔT yields a local maximum of the counts (see Figs. 11–12 of Section 9), and decreasing ΔT by merely 3 minutes would lower the counts by 1 unit and lower the rejection level from 11.1% to 5.2%. These tests rely only on the theoretically secure rate $p = 0.47$.

(2) Test H against A

This test is included here to illustrate the ancillary use of the 33% rate to add evidence that a particular chronology is wrong. Assume that you reject H (correctness of the chronology) if $x \geqslant k$, and thereby accept A (wrongness of the chronology). Then, the probability of falsely rejecting H can be calculated from the binomial distribution appropriate for H ($p = 0.33$). For example, with the 2013 data set for the Low Middle chronology we have 24 misses for +0 and 28 for +1, and we obtain for the probability of falsely rejecting correctness of that chronology 1.47% or 0.04%, respectively. We stay on the conservative side if we pick the lower miss count and the higher P-value (if we had assigned equal probabilities to +0 and +1, we would have taken the average of the two P-values). Thus, we obtain the results seen in Tab. 6.

Chronology	2013; $n = 49$	P $(x \geqslant k)$
–1645	$k = 25$	0.68%
–1637	$k = 24$	1.47%
–1581	$k = 22$	5.51%

Tab. 6 Ammiditana + Ammiṣaduqa. Error probability when rejecting correctness of a chronology.

Thus, for each of the two middle chronologies we can assert, with error probabilities below 2%, that it is wrong. This result is dependent on the reliability of the estimated value $p = 0.33$.

With this test, the case of the Low chronology (–1581) is inconclusive. While in Fig. 6 it sits where we expect a wrong chronology to sit, Tab. 6 shows that the fit is not sufficiently poor that the chronology can be rejected on its own merit with the conventional 5% significance level.

5.2.3 *Combining the Ammiṣaduqa and Ammiditana data by Bayesian methods*

Probabilities from different sources are easiest to combine by Bayesian methods, but it is difficult to agree on the choice of prior probabilities. We must consider eight possibilities: four chronologies, and for each of them absence or presence of an additional intercalation. For the following I give equal probabilities 0.25 to the four main chronologies, and probability α to the presence of an additional intercalation. As in Section 5.1

these priors are multiplied by

$$\left(\frac{p\,(1-q)}{(1-p)\;q}\right)^k$$

where $p = 0.33$, $q = 0.47$, and k is the number of misses. The posterior probabilities of the eight possibilities are obtained by scaling the resulting values so that they sum to 1. The sum over the four components with additional intercalation then gives the posterior probability β of having such an intercalation, and for each particular chronology the posterior probability is the sum of the two values without and with intercalation.

If we were able to prove that there was no additional intercalation, we have $\alpha = 0$. If we could make sure that there was one, we have $\alpha = 1$. Some people might want to formalize ignorance by $\alpha = 0.5$, but most might gravitate towards a small value, say $\alpha = 0.05$ or $\alpha = 0.1$. The results of the calculation for the 2013 set (i.e. with the miss counts of Tab. 5) are listed in Tab. 7.

We note first that $\alpha = 0.1$ *suffices to boost the posterior probability of an intercalation to* $\beta = 0.85$ *and the posterior probability of the High Chronology to 0.91, while the other three chronologies are limited to posterior probabilities below 0.04. Second, confirmation of an additional intercalation (*$\alpha = 1$*) would render the Middle Chronologies utterly implausible.*

The results are quite dependent on the observed miss counts. With the 2010 data set of Tab. 4, the Ammiṣaduqa count is more strongly in favor of the High Chronology, and correspondingly the posterior probability of the latter is as high as 0.91 already for $\alpha = 0$, and the posterior probabilities of the other three chronologies are all below 0.05.

Prior and posterior probabilities of an additional intercalation:

prior α	0.000	0.050	0.100	0.500	1.000
posterior β	0.000	0.731	0.852	0.981	1.000

Posterior probabilities of chronologies:

prior α		0.000	0.050	0.100	0.500	1.000
HC	−1701	0.460	0.843	0.906	0.973	0.983
MC	−1645	0.142	0.038	0.021	0.003	0.000
LMC	−1637	0.256	0.069	0.038	0.005	0.000
LC	−1581	0.142	0.050	0.035	0.018	0.016

Tab. 7 Ammiṣaduqa and Ammiditana data, 2013 miss counts of Tab. 5. Posterior probabilities of the four main chronologies.

5.3 Case 3: Hammurabi-Samsuiluna

For the Hammurabi-Samsuiluna segment I repeated the analysis of 1982,[28] comprising 54 month-lengths, but used newer astronomical programs. The intercalations are highly irregular: from Hammurabi year 32 to 36 the New Year longitude increases by 72°. The consequence is that for each candidate chronology we must consider at least three different seasonal alignments. Some misgivings about possibly misplaced intercalations remain.[29] If we treat the data as independent evidence, the High chronology (–1701) again comes ahead. Its miss rate of $20/54 = 37\%$ is compatible with that of a correct chronology, but unpleasantly high and therefore offers only weak supportive evidence.

5.4 Case 4: Ur III

For the Ur III period the situation is more complex, and I shall discuss some of the problems. If these problems can be resolved, the Ur III month-length data might attain decisive chronological relevance.

By 1986 we had a total of 228 month-lengths. In 2013 this number was modestly increased to 240. What I shall discuss here is my more comprehensive analysis of the smaller earlier set, but using newer programs. Among the different parts of the data, the Drehem segment from Amar-Sin to Ibbi-Sin ($n = 126$) probably is to be trusted most. With the Umma segment from Amar-Sin to Ibbi-Sin ($n = 60$) there are doubts about the intercalations,[30] and with the Šulgi segment from year 39 to 48 ($n = 42$) there are serious doubts about the calendar.

The relative chronology from the beginning of the Ur III dynasty to the end of the Hammurabi dynasty is well established. By reckoning back from the four main Venus chronologies one obtains for Amar-Sin year 1 = –2099, –2043, –2035, –1979.[31] We stay on the safe side by assuming that the true date is within ± 10 years of the back-reckoned dates, and that the New Year syzygy longitude is between 310° and 50°. Then we obtain about 75 feasible alignments for each of the four chronologies: a range of 21 years for the chronology and a little more than 3 months for the season. The four chronologies together give a total of 252 feasible alignments (there is an overlap for the middle chronologies).

A simple calculation with the binomial distribution shows that if we are considering the best of 75 alignments, we need over 70 month-lengths such that the correct chronology has an even chance of sticking out, and if we want it to stick out with 90% probability, we need about 180 correctly distanced month-lengths. In any case, the Umma and

28 Huber, Sachs, et al. 1982.

29 See Huber, Sachs, et al. 1982, 36.

30 See Huber, Sachs, et al. 1982, 38.

31 See Sallaberger 2013, who points out that independent reconstructions suggested uncertainties in the range of ±1 year.

Šulgi segments are too small to be used for independent dating on their own – that is, unless we encounter extraordinarily lucky low miss counts. See also the discussion of the NB data in Section 4.

Among 8000 alignments between –2213 and –1567, the absolutely best fit of the 228 month-lengths gave 84 misses (with the earlier programs 83 misses)[32] and was obtained for three chronologies (Amar-Sin year 1 = –2093, –2020, or –1775). The first is inside a feasible window, corresponding to the High chronology. Among the 252 feasible alignments, by chance the 7 best of them happened to contain representatives from all 4 main chronologies. The 6 alignments matching Middle or Low chronologies had miss counts between 88 and 90.

The best obtained miss number is unpleasantly high ($^{84}/_{228} = 36.8\%$). However, the Drehem subset from Amar-Sin year 1 to Ibbi-Sin year 2 for the same alignment (Amar-Sin Year 1 = –2093, syzygy number of month I $= -13\,516$) gives a miss rate close to the NB value ($^{43}/_{126} = 34.1\%$), and the Umma subset even a lower one ($^{18}/_{60} = 30\%$). On the other hand the Šulgi segment contributed an extraordinarily high number of misses to the total, namely 23 out of 42 months. Note that this rate, $^{23}/_{42} = 54.8\%$, lies even above the rate expected for a wrong chronology, and the probability that a correct alignment produces 23 or more misses is merely 0.3%. But by aligning the Šulgi segment 5 months earlier, the number of misses was reduced from 23 to 14. Through this hypothetical modification the miss rates were reduced to the NB value: namely for the Šulgi segment to $^{14}/_{42} = 33.3\%$ and for the full set to $^{75}/_{228} = 32.9\%$. With this modification, the fit of the –2093 chronology, giving 75 misses, was far superior to the best of the other feasible alignments (86 misses for the –2037 and –1979 chronologies). In any case, the original Šulgi segment appeared to be the odd man out, and I wondered whether Šulgi's years began in fall.

The calendars of Drehem and Umma were not synchronized, and several intercalations differ.[33] The intercalary months usually, but not always, were inserted at the end of the year. Sometimes a 13th month was used by the scribes as a placeholder for the first month of the next year, if the name of the new year was not yet known to them.

Wu Yuhong distinguishes between two different calendars used in Ur III times.[34] The month-names of what he calls the Mašda calendar were *i:* iti-maš-dà-gu$_7$, *ii:* iti-šeš-da-gu$_7$, *iii:* iti-u$_5$-bí-gu$_7$, *iv:* iti-ki-siki-^{d}Nin-a-zu, *v:* iti-ezem-^{d}Nin-a-zu, *vi:* iti-á-ki-ti, *vii:* iti-ezem-^{d}Šul-gi, *viii:* iti-šu-eš-ša, *ix:* iti-ezem-maḫ, *x:* iti-ezem-an-na, *xi:* iti-ezem-Me-ki-gál, *xii:* iti-še-kin-kud. However, at least in Šulgi years 44–48, an alternative so-called Akiti calendar was in use, where the year began in fall, with month *vi:* iti-á-ki-ti of the Mašda calendar. This would seem to give a posterior justification to the experi-

32 See Huber 1987, 14–15.

33 This was noted already by Huber, Sachs, et al. 1982, 38. See now Wu Yuhong 2002 for a detailed investigation.

34 Wu Yuhong 2000.

mental 5-month shift I had applied to the Šulgi data (the years 44–48, where the Akiti calendar had been in use, contain about three quarters of the Šulgi month-lengths available to us). From Amar-Sin on, the Mašda calendar was in use.

Now, what should we do: keep the original Šulgi data, shift them by 5 months, or ignore them? Either way, the Ur III data provide additional, admittedly not quantifiable support for the High chronology. The Middle chronologies give poorer fits. But the Ur III calendars and their synchronization clearly need more investigation before they can be fully trusted for the purposes of dating.

6 Summary: internal consistency and coherence of the results

The Ammiṣaduqa month-length data show the pattern to be expected from one correct and three wrong chronologies, see Figs. 3–4, and they point toward correctness of the High chronology (–1701). The Ammiditana data show the same behavior, see Fig. 5. The discussion of Tab. 5 in Section 5.2 provides strong evidence in favor of the High chronology and against the Middle chronologies. Clean quantitative results are obtained by forming a working hypothesis on the basis of the Ammiṣaduqa data and then testing it with the Ammiditana data. This approach allows to affirm the High chronology on at least the 5% level, and if we are willing to lower ΔT by 3 minutes against the arbitrarily assumed default formula, it is confirmed even on the 1% level. By a Bayesian argument it can be shown that the High chronology is roughly 25 times more probable than each of the other three main chronologies (Tab. 7).

The Hammurabi-Samsuiluna and the Ur III data support these results, even if their reliability might be questioned. In addition, the Simānu eclipse of EAE 20, commonly thought to refer to the death of Šulgi, can be identified with the lunar eclipse of –2094 July 25, just one year before the date –2093 of Amar-Sin year 1 suggested by the month-lengths. None of the other possible identifications of that eclipse fall within one of the time windows implied by the Venus chronologies.[35]

The test performed in Section 5.2.1 implies that also for Old-Babylonian times the miss rate for a correct chronology is substantially below that for a wrong chronology. The miss counts corresponding to the High chronology, as shown in Figs. 3, 4, 5, and 6, all approximately correspond to the 33% Neo-Babylonian rate. This does not prove that the OB rates for correct chronologies are equal to the NB rates, but at least they do not contradict such an assumption.

35 See Huber 1999/2000, 77, for a list of alternative identifications (the next eclipses matching the description in the omen are in –2018, –2007, –2001, –1936).

In my opinion, internal coherence of the results is an even stronger indication of their trustworthiness than any statistical significance assertions. And anyone desiring to defend one of the Middle chronologies, rather than ignore the opposing month-length evidence, or simply discount it as being the only witness in favor of the High chronology, should better find plausible arguments discrediting the evidence of Figs. 3, 4, 5, and 6. In order to be convincing, such arguments would seem to require new data. They might be based on a large, reliable set of contradictory new month-length data, or on new eponym lists bridging the interval between Old Assyrian and Neo-Assyrian times.

7 Modeling and simulation of crescents and month-lengths

In statistics, the principal purpose of modeling and stochastic simulation quite generally is to obtain crude estimates of the statistical variability of various empirical measurements. The models are designed to give a satisfactory phenomenological description of the situation. Whether they can give a causal explanation is a more difficult and possibly unanswerable question. Here are the facts and assumptions on which we shall base the models.

For randomly chosen wrong chronologies the agreement/disagreement rates between observed and calculated month-lengths are fixed by astronomical theory, that is by the length of the synodic month (29.5306 days): 53% of the months have 30 days, 47% have 29 days. It follows that a collection of 30-day months, when aligned at random along a calculated sequence, has an expected miss rate of 47%.

An approximate estimate of the variability of empirical miss rates then can be obtained from the binomial distribution for which the miss counts have the standard deviation $\sqrt{n\,p\,(1-p)}$. An alternative, perhaps more reliable version can be found by aligning a batch of observed 30-day months at many positions along a calculated sequence of such months. See Fig. 2 for a comparison between the two approaches.

For a correct chronology the LB astronomical texts give a rock bottom lower limit of about 10% for the rate of discrepancies between observed and calculated month-lengths. I used the calculated sequence of 33 000 months to check the effects of the pure gray-zone model (with $d = 1.1^\circ$). The probability of seeing the crescent 1 day early was 2.56%, and that of seeing it 1 day late was 2.51%. Months never were shortened to 28 days by gray-zone effects, but occasionally they were lengthened to 31 days. With calculated 29-day months, lengthening to 31 days happened in 0.06% of the cases, with 30-day months in 0.23% of the cases (that is, about once in a human life-time). Calculated 29-day months were lengthened to 30 days in 10.4% of the cases, and 30-day months were shortened to 29 days in 9.4% of the cases. This corresponds to the 10% miss rates of the LB astronomical texts.

The NB administrative texts give a higher discrepancy rate. As mentioned in Section 4 there are 149 NB texts dated to day 30, and 49 of them occur in a month that according to calculation has 29 days. The discrepancy rate thus is approximately $^{29}/_{149} =$ 33%. Up to 27 of these 49 discrepancies might be explainable by 'gray-zone' effects of early or late sightings, namely those for which $|\Delta h| \leqslant 1.1°$, but at least 22 discrepancies must have a different cause. Note that at the begin of a month lengthening can occur only because of a fortuitous early sighting in the range $-1.1 \leqslant \Delta h \leqslant 0$, while at the end poor weather or sheer lack of care might cause a delay with values of Δh larger than $+1.1$.

I propose a simple two-component model. One component corresponds to the 'gray-zone' model of the astronomical texts, and the other to a practice of 'overhang dating' or 'double dating' (a term preferred by Michael Roaf) by the ancient scribes: when dating a text they would occasionally write day 30 in cases where they more properly should have written day 1 of the next month. The consequences of such a model shall be developed in Section 8 (following next). Evidence for the presence of overhang dating is contained in Sallaberger's remark,[36] according to which Amar-Sin year 5 contained 9 day-30 dates, instead of the expected 6. In other words, we can have more day-30 dates than are astronomically possible. Note that variability caused by 'gray-zone' effects would stay on in the calendar, while 'overhang' effects would not. If the officials responsible for the calendar should decide that the preceding month had had only 29 days, the scribe simply would skip a day and let day 30 be followed by day 2.

A letter to an Assyrian king (presumably Assurbanipal) has the remarkable passage:

> I observed the (crescent of the) moon on the 30th day, but it was high, too high to be (the crescent) of the 30th. Its position was like that of the 2nd day. If it is acceptable to the king, my lord, let the king wait for the report of the Inner City before fixing the date.[37]

This letter is interesting because it shows that the beginning of the month could be fixed retroactively, and possibly the length of the preceding month even could be shortened to 28 days.

A possible argument against this simple overhang model is that a (preliminary) investigation of Ur-III-time month-lengths based on regular deliveries did not seem to give a substantially better agreement with calculation than those based on day-30 dates.

We do not know when and why the scribes would use overhang dating, but we can crudely estimate how often it may have occurred in the NB material. In Section 4, I had

36 Sallaberger 1993, 12.

37 Parpola 1993, Letter 225.

estimated that overhang might occur between 40% and 50% of the cases. The theoretical model of the next section gives the best fit when assuming an overhang probability $p_{ov} = 0.46$.

Overhang dating according to the model just described raises a problem: if many scribes independently use it, then every true hollow month ultimately will acquire some overhang dates. But true full months will obtain day-30 dates more often than true hollow months. In this case the proper solution is to count dates with their observed multiplicity. On the other hand, whenever multiple dates originate in the same scribal office, they are strongly dependent and one should count them only once. It is difficult to separate these cases. Here, I acted as if the second case applied and counted multiple occurrences only once (with the presently available material they are relative rare anyway).

8 Theoretical miss rates

Independently of the cause of the discrepancies between calculated and observed month-lengths, the calculation of the miss rates of day-30 dates is, essentially, a straight exercise with conditional probabilities.

The miss rate in question is the conditional probability, given a recorded day-30 date (D30), that the underlying month calculates as a 29-day month (C29):

$$p_{\text{miss}} = \mathrm{P}(\mathrm{C29}|\mathrm{D30}) = \frac{\mathrm{P}(\mathrm{C29}\ \&\ \mathrm{D30})}{\mathrm{P}(\mathrm{D30})}.$$

We have

$$\mathrm{P}(\mathrm{C29}\ \&\ \mathrm{D30}) = \mathrm{P}(\mathrm{D30}|\mathrm{C29})\,\mathrm{P}(\mathrm{C29}),$$
$$\mathrm{P}(\mathrm{D30}) = \mathrm{P}(\mathrm{D30}|\mathrm{C29})\,\mathrm{P}(\mathrm{C29}) + \mathrm{P}(\mathrm{D30}|\mathrm{C30})\,\mathrm{P}(\mathrm{C30}).$$

Here

$$\mathrm{P}(\mathrm{C29}) = 0.47,$$
$$\mathrm{P}(\mathrm{C30}) = 0.53.$$

If we assume zero width for the gray zone, and that overhang occurs at random with probability p_{ov}, and if we assume that dates higher than day 30 are not permitted, then

$$\mathrm{P}(\mathrm{D30}|\mathrm{C29}) = p_{ov},$$
$$\mathrm{P}(\mathrm{D30}|\mathrm{C30}) = 1,$$

and we can substitute these values into the above formula for p_{miss}.

Now assume a gray zone with finite width, such that in the absence of overhang the miss rate for month-lengths is $\mu = 0.1$.

Then, the last two probabilities are changed to

$$\mathrm{P}(\mathrm{D30}|\mathrm{C29}) \;=\; p_{\mathrm{ov}} + (1 - p_{\mathrm{ov}})\,\mu,$$
$$\mathrm{P}(\mathrm{D30}|\mathrm{C30}) \;=\; 1 - (1 - p_{\mathrm{ov}})\,\mu.$$

The justification for these formulas is as follows.

– Given that there is overhang in that particular month, then $\mathrm{P}(\mathrm{D30}|\mathrm{C29}) = 1$, and given that there is none, then $\mathrm{P}(\mathrm{D30}|\mathrm{C29}) = \mu$.
  If overhang occurs with probability p_{ov}, then we obtain the first of the above formulas.
– Given that there is overhang in that particular month, then $\mathrm{P}(\mathrm{D30}|\mathrm{C30}) = 1$, and given that there is none, $\mathrm{P}(\mathrm{D30}|\mathrm{C30}) = 1 - \mu$.
  If overhang occurs with probability p_{ov}, then we obtain the second of the above formulas: $\mathrm{P}(\mathrm{D30}|\mathrm{C30}) = p_{\mathrm{ov}} + (1 - p_{\mathrm{ov}})(1 - \mu) = 1 - (1 - p_{\mathrm{ov}})\,\mu$.

I believe the most questionable assumption in the above arguments is that overhang occurs at random (i.e. independent of Δh). It appears at least plausible that overhang is more likely to occur for small values of Δh than for large ones. But the NB material does not really support such a conjecture, see Tab. 3. It shows a clear cluster of values in the gray zone ($\Delta h \leqslant 1.1$). In the range between 1.6 and 6.8 the number of Δh values shows a moderate decrease, but this decrease seems to go in parallel with a decrease in the number of Δh values calculated for 29-day months.

The following probabilities were calculated with the above model, on the basis of 33 000 calculated month-lengths, not on the binomial distribution, by applying the model to a large number of randomly chosen subsets of the calculated sequence. The overhang probability was empirically adjusted to $p_{\mathrm{ov}} = 0.46$, so that the combined model approximately reproduced the miss rate of $50/153 = 0.327$ of the Neo-Babylonian material.

For the purpose of these modeling calculations I assumed for the gray zone model:

– if $\frac{\Delta h}{d} < -1$, the crescent is never seen;
– if $-1 \leqslant \frac{\Delta h}{d} \leqslant 1$, the crescent is seen with probability $(1 + \frac{\Delta h}{d})/2$;
– if $\frac{\Delta h}{d} > 1$, the crescent is always seen,

with $d = 1.1°$ (in 1982 I had used $d = 1.0°$).

Pure overhang model (overhang probability $p_{\mathrm{ov}} = 0.46$):
  Miss rate $\mathrm{P}(\mathrm{C29}|\mathrm{D30}) = 0.290$

Pure gray zone model (zone width $d = 1.1°$):
  $\mathrm{P}(\mathrm{D30}|\mathrm{C29}) = 0.104$
  $\mathrm{P}(\mathrm{D30}|\mathrm{C30}) = 0.906$
  Miss rate $\mathrm{P}(\mathrm{C29}|\mathrm{D30}) = 0.092$

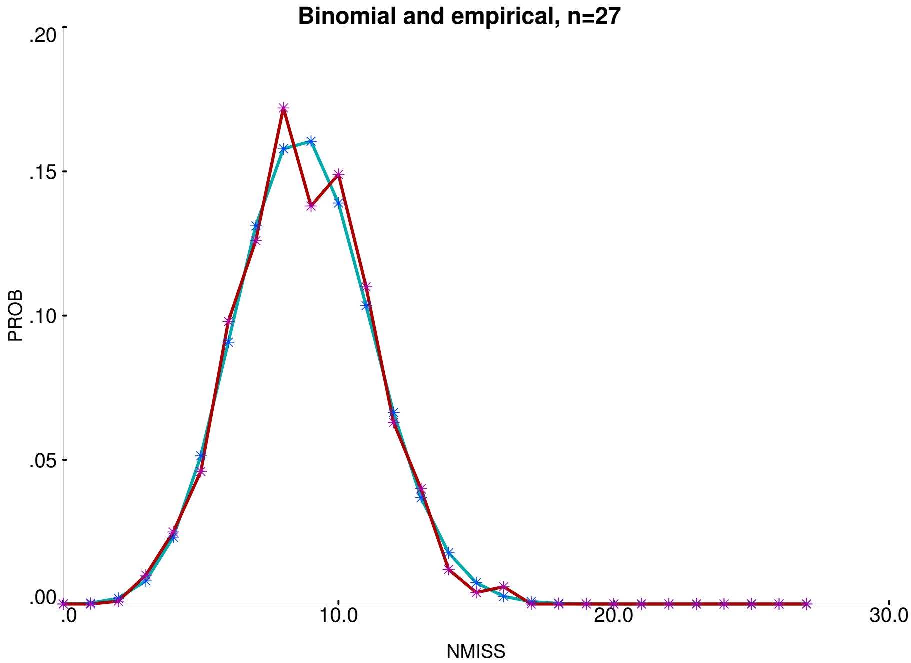

Fig. 7 Comparison between the binomial distribution ($n = 27$, $p = 0.325$, blue) and the empirical frequencies obtained from 1000 samples based on the overhang model ($d = 1.1°$, $p_{ov} = 0.46$, red).

Combined model ($p_{ov} = 0.46$, $d = 1.1°$):

$P(D30|C29) = 0.516$

$P(D30|C30) = 0.949$

Miss rate $P(C29|D30) = 0.325$

The sequence of calculated month-lengths is not quite random, and therefore the distribution of the miss counts does not necessarily follow a binomial distribution. In the case of the wrong chronologies, it had been possible to compare the binomial distribution with the results of a large number of alignments (Fig. 2). The case of the correct chronology is less straightforward, but we can compare the binomial distribution with the results of the simulated error model. Also here the binomial distribution gives a very good approximation. Fig. 7 shows a comparison between the empirical frequencies based on the above overhang model ($d = 1.1°$, $p_{ov} = 0.46$) and the binomial distribution ($n = 27$, $p = 0.325$). The empirical frequencies were obtained by drawing 1000 random samples of size 27 from the calculated sequence of months and then applying random gray zone and overhang effects.

9 Sensitivity to the clock-time correction ΔT

A central problem of historical astronomy is our insufficient knowledge of the clock-time correction ΔT = ET − UT (the difference between the uniform time scale ET underlying the astronomical calculations and civil time UT depending on the irregular rotation of the earth). Because of tidal friction ΔT increases quadratically with time, but it is subject to sizable random fluctuations. By now, it is reliably known back to 700 BC within a standard error of approximately 5 minutes, but its extrapolation from there to 2000 BC is affected by a standard error of about 1 hour.[38]

For the present paper I have assumed a formula proposed by Morrison and Stephenson (in a paper published in 1982) as my default:[39] $\Delta T = c\,t^2$ sec, with $c = 32.5$ and t measured in centuries since AD 1800, together with lunar orbital acceleration $\dot{n} = -26''/cy^2$. I made this choice for three reasons: first, because of its simplicity, second, because calculations based on it agree very closely with the traditional tables by P. V. Neugebauer and Tuckerman,[40] and third, last but not least, if the solar eclipse of Sargon of Akkad has been correctly identified, it implies that ΔT in the mid-24th century was between −20 and +7 minutes of that default.[41] For that time this corresponds to a range of c between 31.7 and 32.8. Moreover, this solar eclipse now would seem to determine the clock-time correction ΔT with a standard error of the order of 10–15 minutes back to the 24th century BC. If the Ugarit eclipse of –1222 has been correctly identified, perhaps 10 minutes higher values of ΔT, corresponding to values of c that are about 0.5 higher, may hold for the Ur III and OB periods.[42]

I found it convenient to vary ΔT by modifying the parameter c; note that a change of 1 unit in c by –1700 amounts to a change of 20 minutes in ΔT. I believe that the most probable range of c is between 31.5 and 33.5, but for the sensitivity study depicted in Figs. 8, 9, 10, 11, and 12, I have applied a range of c between 27 and 38, that is of approximately ±2 hours for the Ur III and OB periods. These figures are based on the data available in 2013.

One of the questions to be addressed by this sensitivity study was whether perhaps the sensitivity of the miss counts to ΔT was such that that ultimately they might be used to improve our estimates of ΔT.

38 Huber 2006.

39 Morrison and Stephenson 1982.

40 P. V. Neugebauer 1929; Tuckerman 1962; Tuckerman 1964.

41 Huber 2012; Morrison and Stephenson 1982. The formula more recently proposed by the latter authors in 2004 for extrapolation beyond –700, namely $\Delta T = -20 + 32t^2$ sec, with t in centuries since AD 1820, differs only negligibly: the difference in ΔT increases from 0 in –600 to 3 minutes in –1700 and 6 minutes in –2400 (Morrison and Stephenson 2004).

42 Huber 2012, Fig. 3, showing the deviations from the default ΔT and their variability.

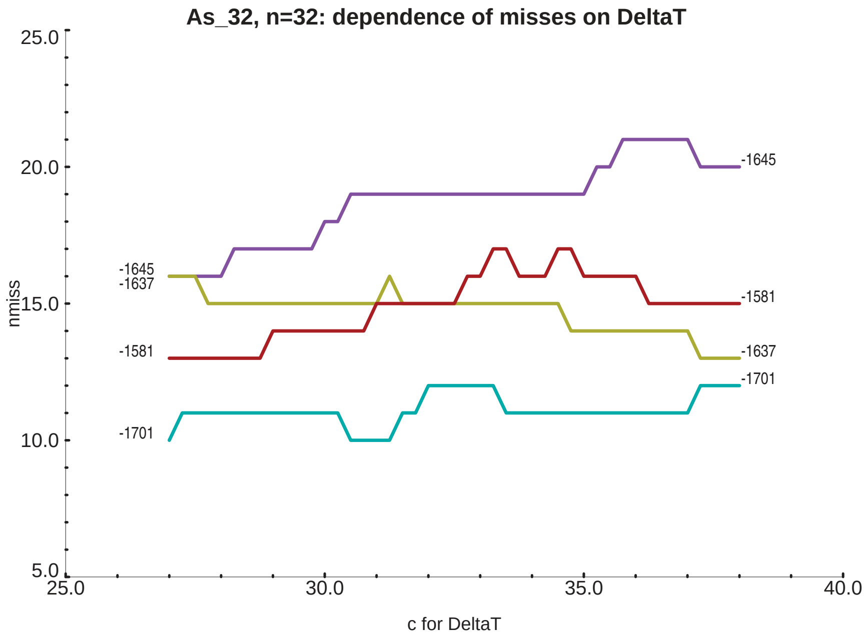

Fig. 8 Sensitivity of miss counts to ΔT, Ammiṣaduqa data; $n = 32$.

We note that in the figures for the Ammiṣaduqa data (Fig. 8), for the Ammiditana (+1) data (Fig. 10), and for their combination (Fig. 12), the default ΔT ($c = 32.5$) yields a local maximum of the miss counts for the High chronology, with a local minimum for c between 30.5 and 31.25. This minimum is reached by a decrease in ΔT of about 25 minutes. I was almost tempted to derive an improved estimate of ΔT for the OB period from this. At the same time, lowered values of the miss counts would much improve the rejection levels in Section 5.2, see in particular Section 5.2.1. However, I do not intend to insist on these arguments.

But in any case, this sensitivity study shows that our default choice for ΔT, by leading to a local maximum for the miss counts, happens to be conservative.

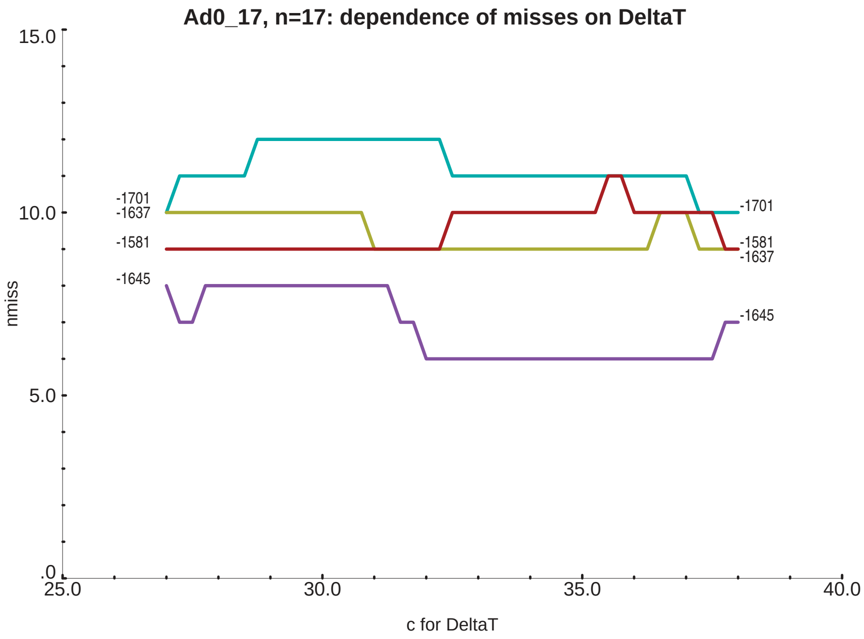

Fig. 9 Sensitivity of miss counts to ΔT, Ammiditana data, +0 intercalation; $n = 17$.

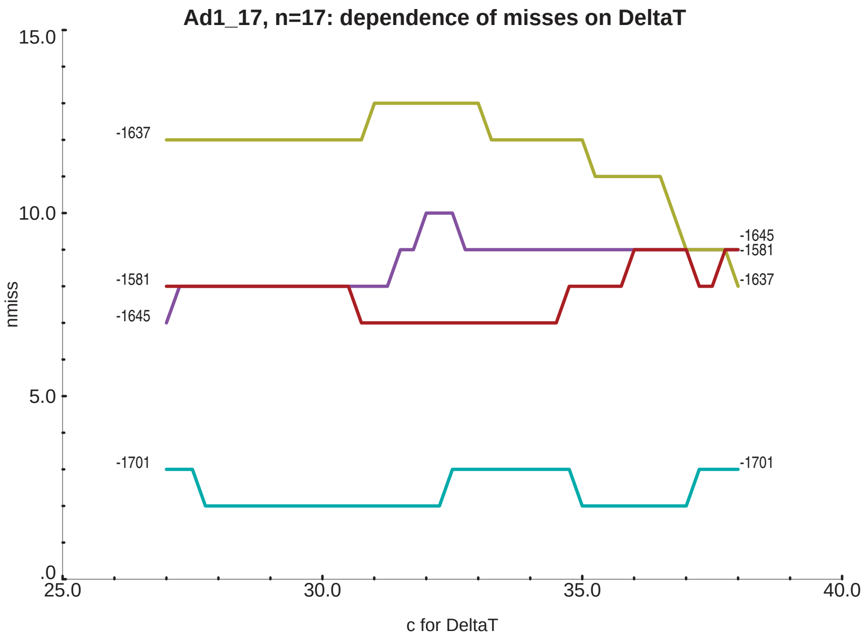

Fig. 10 Sensitivity of miss counts to ΔT, Ammiditana data, +1 intercalation; $n = 17$.

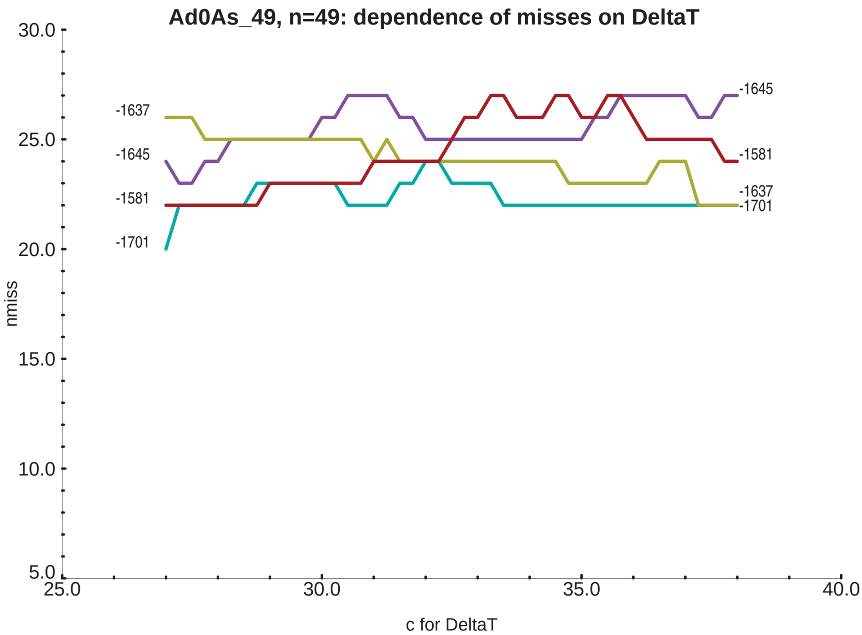

Fig. 11 Sensitivity of miss counts to ΔT, Ammiditana-Ammiṣaduqa data, +0 intercalation; $n = 49$.

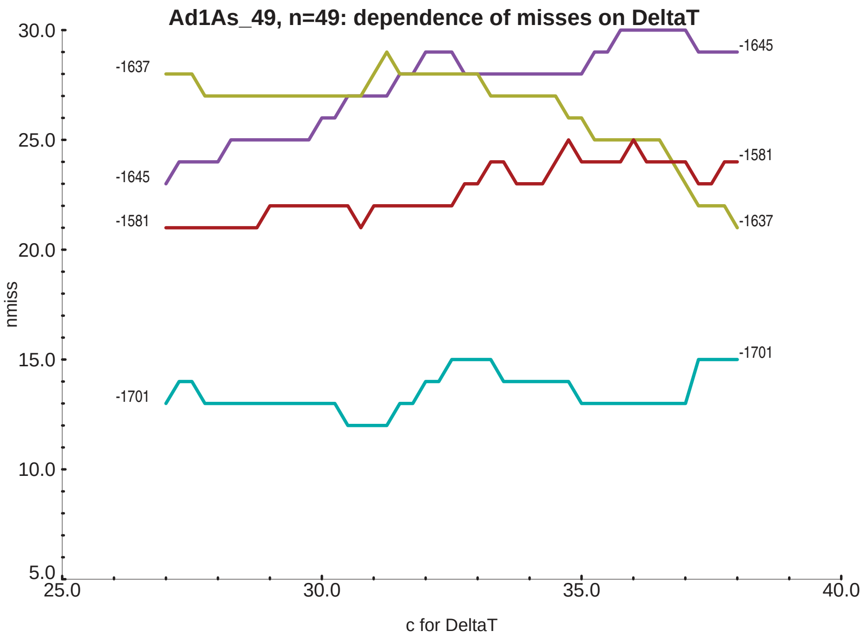

Fig. 12 Sensitivity of miss counts to ΔT, Ammiditana-Ammiṣaduqa data, +1 intercalation; $n = 49$.

10 Appendix: the underlying data base

10.1 Intercalations during the reign of Ammiṣaduqa

The connection of the Venus text with the reign of Ammiṣaduqa had been established by Kugler in 1912, when he identified the year name 'Year of the Golden Throne' occurring in the 8th year of the Venus text with the name of the 8th year of king Ammiṣaduqa.[43] Since then, some doubts about the conclusiveness of the identification have been voiced (there are other year names involving a Golden Throne), but we now can establish the connection beyond doubt with the help of the intercalations.

The Venus text has first visibilities of Venus in the morning of Year 1 XI 18 and of Year 17 XII 14, thus spaced 16 years and 1 month. Between these dates there are 10 synodic periods of Venus, corresponding to 5840 days or 198 synodic months. As 16 lunar years contain only $16 \times 12 = 192$ months, there must be 5 intercalations between these two dates. In order to obtain the correct spacing between Venus phenomena, the 5 intercalations in question must have occurred as:

(1) (4A or 4U), (5U), (9A or 10U), (11U attested), (13U or 13A or 14U).

Here, A stands short for a second Addāru (XII_2), U for a second Ulūlu (VI_2).[44] On the other hand, contemporary administrative documents attest the following intercalations for the first 16 years of Ammiṣaduqa:

(2) 4A, 5U, 10U, 11U, 13A.

In addition, they attest a 17A. Note that texts from Sippar Amnanum show that the years previously provisionally denoted $17 + a$ and $17 + b$ can be identified with the years 17 and 18.

The probability that an agreement as good as that between (1) and (2) occurs by chance is less than 1 in 1000. This can be calculated as follows. There are $\frac{15\times 14\times 13\times 12\times 11}{6\times 2}$ $= 30\,030$ possibilities for placing 3 U tokens and 2 A tokens in 15 slots (the 15 years from 2 to 16). However, not all are feasible. Intercalations are inserted to keep the years in step with the agricultural seasons, and on average a regular year decreases the New Year longitude by 10.7°, while an intercalary year increases it by 18.4°. If we only permit intercalation patterns that keep the difference between the maximal and minimal New Year longitude below 45° or 50° – for the actual Ammiṣaduqa intercalations this difference is 44.5° – merely between 20% or 30% of the possibilities remain feasible. Among the 12 patterns of intercalations made possible by the Venus text, 4 satisfy the requirement

43 Kugler 1912.

44 Following the convention of Parker and Dubberstein 1956, 6.

that there are 3 U and 2 A tokens. Thus, the probability of hitting by chance a pattern compatible with the Venus text is approximately 4 in 6000 trials, that is 0.0007.

This has important consequences. It shows that the Venus text refers to the time of Ammiṣaduqa, and that the traditional year count of the Venus tablet agrees with years 1 to 17 of that king. Moreover, we know that we have a complete list of all intercalations of years 1 to 17, except that perhaps an intercalation 1U might be missing.

Tab. 8 lists the intercalations attested or implied by the Venus Tablet, and those attested by contemporary contracts. Unpublished data mentioned in LFS are highly unreliable; among them, 5U has now been confirmed by the Cornell text CUSAS 8 55,[45] while 14U in all probability is wrong. The second-but-last column counts the number of months preceding the beginning of the year, and the last column gives the deviation of the New Year syzygy longitude from that obtained for Year 5 (which for all chronologies within 1° corresponds to the median value).

Year 18 is missing in the Venus text, and years 19–21 constitute the highly questionable 'Section III' of the text. We shall ignore evidence derived from that section of the Venus text.

Seth Richardson points out that the names of years 13 and 17 are almost indistinguishable, so some texts may have been misclassified. This should not create problems with regard to intercalations (both years have a second Addāru), but might do so with regard to month-lengths.

10.2 Intercalations during the reign of Ammiditana

The intercalations in Tab. 9 are attested for the 37 years of Ammiditana.[46] The last column gives an arbitrary count of month numbers preceding the begin of the year (as in Section 10.1 above).

For the first 21 years of Ammiditana only 4 or 5 intercalations are attested, whereas the expected average is 7 in 19 years, so some appear to be missing. For the last 16 years (years 22 to 37) 8 intercalations are attested. There is a surprising sequence of 4 consecutive intercalary years (25 to 28), and even if we delete the improbable month XI_2(!?) in year 25, we are still slightly above the expected average. For 25 XI_2 the text has ITI ZIZ_2 DIRI(= SI.A) instead of the expected ITI ZIZ_2.A, and we can assume that this is a scribal error. So it is possible that we have the full pattern of intercalations for the years 22 to 37.

45 Search for this text in http://www.archibab.fr/4DCGI/recherche5.htm, under Mois: 6-bis, Roi: Ammiṣaduqa, Année: 5 (visited on 17/7/2017).

46 From Huber, Sachs, et al. 1982.

Year	Venus Tablet	Contracts	MNU	NYL
1			0	12°
2			12	2°
3			24	–8°
4	A or U implied	A VAS 7 76; BM 17563	36	–18°
5	U implied	U CUSAS 8 55	49	0°
6			62	18°
7			74	7°
8			86	–4°
9	9A or 10U		98	–14°
10	9A or 10U	U YOS 13 532; BE 6/1 106; BM 81130; BM 26602	110	–24°
11	U attested	U CT 8 3a; BM 81350	123	–6°
12			136	12°
13	13U or 13A or 14U implied	A YOS 13 404; TLB 1 211; BIN 7 208-9; BM 78461; BM 79435; BM 81396; BM 81747; Dalley, Edinb. No.20; OLA 21 no.69; CUSAS 8 13	148	1°
14		(U LFS unpublished)	161	19°
15			173	9°
16			185	–2°
17		A TCL 1 171; BAP 9; VAS NF II 99; YOS 13 53; BM 79010	197	–13°
18			210	5°
19	U attested	U YOS 13 146	222	–5°
20	A or U implied		235	13°
21				

Tab. 8 Intercalations attested or implied by Venus Tablet and those attested by contemporary contracts. Previously uncertain year names: $17 + a = 17$; $17 + b = 18$; $17 + c = 2$; $17 + d = 19$. For $17 = 17 + a$ and $18 = 17 + b$, see Nahm 2014.

Yr	Type	Text	MNU
1	regular	(cf. BE 6/1 82:14–22. Between Ae 28 III 30 and Ad 5 XII 30 there are 5 years and 10 months.)	
2	regular		
3	regular		
4	XII_2	BE 6/1 91; YOS 13 205	
5			
6			
7			
8			
9			
10	XII_2	PBS 8/2 202; AO 8126; BM 17313; BM 78465	
11	(XII_2	LFS unpubl.)	
12			
13	XII_2	BM 22522	
14	XII_2	YOS 13 1 = HSM 48 (coll. Moran)	
15			
16			
17			
18			
19			
20			
21			
22	XII_2	YOS 13 197; PSBA 34 24; see YOS 13 p.1 and JCS 13 39a	799
23			812
24			824
25	XI_2(!?)	YOS 13 272	836
26	XII_2	CT 6 39a = BM 80596; BM 16684(?, coll. Walker)	848
27	XII_2	BE 6/2 109	861
28	XII_2	BM 80977	874
29			887
30			899
31			911
32	XII_2	BM 78668; BM 16535; MHET 1/1 11	923
33	XII_2	BE 6/2 112	936
34			949
35			961
36			973
37	XII_2	Kraus, Edikt 28 3′; RA 63,48 37–39; YOS 15 72; BM 79897	985

Tab. 9 Intercalations attested for the reign of Ammiditana.

However, the intercalary pattern is highly irregular. There are 3 consecutive intercalations in the years 26–28. I do not think that in the immediately preceding or following 3 years intercalations are missing, but for the 7 years between Ammiditana 34 and Ammiṣaduqa 3 only a single intercalation is attested. Note that on average a regular year decreases the New Year solar longitude by 10.7°, while an intercalary year increases it by 18.4°. Thus, the 3 intercalary years 26–28 increase the New Year longitude by 55°, while the 7 years from year 34 on, containing 6 regular and 1 intercalary years, decrease it by 46°.

Given the irregular pattern of intercalations, the lack of attested intercalations between the years 15–21, and the wide spread of the New Year longitudes (their range is 58° for Ammiditana, 44° for Ammiṣaduqa), we cannot exclude the possibility that near the border between the Ammiditana and Ammiṣaduqa blocks an unattested intercalation is missing, for example a XII_2 in Ammiditana Year 36, or a VI_2 in Ammiṣaduqa Year 1. This choice shifts the entire block of attested month-lengths (from 24 IV to 36 XII) together by 1 month. We should keep the possibility of an additional intercalation in mind.

10.3 The Ammiṣaduqa month-lengths

The month-lengths in Tab. 10 are attested in contracts from the reign of Ammiṣaduqa, years 1–19.

The list is taken from *Astronomical Dating of Babylon I and Ur III*,[47] 21 months with 11 later additions (6 from Marten Stol, between 1982 and 2010, and 5 from Seth Richardson in 2013, the latter with superscript R and noted with n for 'new' in the last column). Later on, too late to be used in the calculations, Michael Roaf supplied a table with month-lengths (mostly collated by Frans van Koppen); it was merged into the list in Tab. 10. For the 32 entries used in the calculations, the MNU column gives the month number, arbitrarily counted from month I of Year 1 (assuming that year 1 is regular).

Roaf noted: BM 92520 (Meissner BAP 107) and CBS 01346 (Van Lerberghe Mél De Meyer 159–168) are duplicates. VS 7 109 is dated As 16-02-08 but line 3 mentions iti $bara_2$-zag-gar u_4-30-kam as the date of a recent expenditure. Old Babylonian legal and administrative texts from Philadelphia by Karel van Lerberghe. Note that MLC 1517 (YOS 13 65) has 30 written on top of 2 or vice versa (Year 19 X 2/30) and so is not included in the list in Tab. 10.

47 Huber, Sachs, et al. 1982, 65.

Year	Month	Day	Texts	MNU	
1	VII	30	MAH 16218 (TJDB p.31)	7	
	VIII	30	VAT 06253 (VS 7 68)	8	
	XII	30	BM 78640; BM 79869^{R}	12	
3	IV	30	BM 92606	28	
	VI	30	VAT 06380A (VS 7 73)	30	
4	XI	30	BM 26350a	47	
	XII_2	30	VAT 06238 (VS 7 76)	49	
5	VIII	30	BM 16644^{R}	58	n
	XII	30	MLC 1349 (YOS 13 165)	62	
6	VI	30	BM 80804	68	
7	XII	30	MLC 0452 (YOS 13 126)	86	
11	II	30	BM 80984^{R}; BM 97370^{R}	125	
	VII	30	BM 97733^{R}	131	n
	IX	30	BM 97623 (De Graef, AuOr 20 82f. no. 06)		
12	IV	30	IM 50423 (Edzard, ed Der no.49)	140	
	VII	30	BM 80896^{R}	143	n
	VIII	30	BM 81105	144	
13	I	30	BM 97250^{R}	149	n
	II	30	BM 17146	150	
	VI	30	IM 81586 (Van Lerberghe, Mél. Tanret, 592-594)		
	X	30	CBS 01219		
	XII	30	MLC 0828 (YOS 13 220); BM 81677^{R}; BM 97495^{R}	160	
	XII_2	30	BM 78459^{R}, BM 81096, CBS 01473 (Van Lerberghe, OB Legal 069)	161	n
14	IV	30	BM 79287	165	
	VI	30	Strasbourg 324 (Frank 28)	167	
	VIII	30	CBS 01734 (JCS 11 p.93)	169	

(continued on next page)

(continued from previous page)

Year	Month	Day	Texts	MNU
15	II	30	MLC 0822 (YOS 13 221)	175
	X	30	BM 13596 (RA 69 p.188)	183
	XII	30	BM 80167 (CT 2 18)	185
16	I	30	BM 92520 (Meissner BAP 107); VAT 06382 (VAS 7 109); CBS 01346 (Van Lerberghe, Mél. De Meyer, 159–168)	186
	XI	30	CBS 01672 (PBS 14 pl.64 no.1078); VAT 05925, 05938 (Kugler, SSB II p.246)	196
	XII	30	VAT 05391 (VS 7 121); BM 97495	197
17	XI	30	VAT 06287 (VS 7 133)	208
	XII	30	VAT 06224 (VS 7 139); BM 80404 (CT 48 76)	209
18	III	30	BM 87292 + 87337	
	V	30	BM 81624 (CT 48 78)	215
	X	30	CUNES 51-01-045 (CUSAS 8 40)	
19	III	30	BM 81079[R]	

Tab. 10 Month lengths attested in contracts from the reign of Ammiṣaduqa.
Previously uncertain year names: $17+a=17$; $17+b=18$; $17+c=2$; $17+d=19$.

10.4 The Ammiditana month-lengths

The month-lengths in Tab. 11 are attested in contracts from the reign of Ammiditana.

In view of the incomplete list of intercalations, only month-lengths from the years 22–37 are usable. The list is taken from *Astronomical Dating of Babylon I and Ur III*:[48] 13 months, plus 4 from Richardson 2013, the latter with superscript R and noted with n in the last column. Later on, too late to be used in the calculations, Michael Roaf supplied a table with month-lengths (mostly collated by Frans van Koppen, but in particular the 26.VI and 26.IX and 36.I need to be confirmed); it was merged into the list in Tab. 11. For the 17 entries used in the calculations, the MNU column gives an arbitrary count of month numbers.

48 Huber, Sachs, et al. 1982, 64–65.

Year	Month	Day	Texts	MNU	
1	X	30	VAT 6655 (VAS NF 2 15)		
	XII	30	BM 80336; BM 81465; Bu 1891-05-09, 0473 (CT 6 26b)		
2	XI	30	Di 720 (K. van Lerberghe)		
	XII	30	BM 17482; BM 80623		
4	VIII	30	BM 78704 = ? (CT 33 47b)		
	XII_2	30	CBS 0723 (BE 6/1 91)		
5	XII	30	CBS 0110 (BE 6/1 82)		
6	IV	30	BM 80161 (CT 45 46)		
7	XII	30	U.7183 (UET 5 518); MLC 00452 (YOS 13 126); CBS 0125		
14	III	30	BM 78182 (CT 45 48)		
	IX	30	BM 109169		
	XII_2	30	HSM 48 (YOS 13 1)		
24	I or V	30	BM 81569		
	IV	30	BM 80513	828	n
	VII	30	AO 01679 (TCL 1 153)	831	
	VIII	30	BM 80513		
26	VI	30	VAT 5912 (Kugler, SSB II p.246)	854	
	IX	30	VAT 5806 (Kugler, SSB II p.246)	857	
27	VII	30	BM 97441		
	XII_2	30	CBS 0366 (BE 6/2 109)	874	
29	II	30	MLC 1291 (YOS 13 254)	889	
30	IV	30	TJAUB pl.39(H 31)	903	
	VII	30	BM 97013^R	906	n
31	II	30	CBS 1241 (BE 6/1 83)	913	
	XII	30	CBS 1512 (BE 6/1 84)	923	
32	VIII	30	BM 96990^R	931	n
	XII	30	BM 78609	935	
33	IX	30	BM 97447^R	945	n
34	VIII	30	MLC 0440 (YOS 13 79)	957	
	IX	30	VAT 6392 (VS 7 60)	958	
36	I	30	VAT 06258		
	II	30	MLC 0425 (YOS 13 57)	975	
	XII	30	BM 78719	985	
37	IV	30	BM 97057		

Tab. 11 Month lengths attested in contracts from the reign of Ammiditana.

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